DETERMINANTS

1. If $\Delta_k = \begin{vmatrix} k & 1 & 5 \\ k^2 & 2n+1 & 2n+1 \\ k^3 & 3n^2 & 3n+1 \end{vmatrix}$, then $\sum_{k=1}^{n} \Delta_k$ is equal to

a) $2\sum_{k=1}^{n} k$ b) $2\sum_{k=1}^{n} k^2$ c) $\frac{1}{2}\sum_{k=1}^{n} k^2$ d) 0

2. The solutions of the equation $\begin{vmatrix} x & 2 & -1 \\ 2 & 5 & x \\ -1 & 2 & x \end{vmatrix} = 0$, are

a) $3, -1$ b) $-3, 1$ c) $3, 1$ d) $-3, -1$

3. The value of $\begin{vmatrix} 441 & 442 & 443 \\ 445 & 446 & 447 \\ 449 & 450 & 451 \end{vmatrix}$ is

a) $441 \times 446 \times 4510$ b) 0 c) -1 d) 1

4. If $f(\alpha) = \begin{vmatrix} 1 & \alpha & \alpha^2 \\ \alpha & \alpha^2 & 1 \\ \alpha^2 & 1 & a \end{vmatrix}$, then $f(\sqrt[3]{3})$ is equal to

a) 1 b) -4 c) 4 d) 2

5. If a, b, c are respectively the pth, qth, rth terms of an AP, then $\begin{vmatrix} a & p & 1 \\ b & q & 1 \\ c & r & 1 \end{vmatrix}$ is equal to

a) 1 b) -1 c) 0 d) pqr

6. The minors of -4 and 9 and the cofactors of -4 and 9 in matrix $\begin{bmatrix} -1 & -2 & 3 \\ -4 & -5 & -6 \\ -7 & 8 & 9 \end{bmatrix}$ are respectively

a) $42, 3, -42, 3$ b) $-42, -3, 42, -3$ c) $42, 3, -42, -3$ d) $42, 3, 42, 3$

7. If α, β, γ are the cube roots of unity, then the value of the determinant $\begin{vmatrix} e^{\alpha} & e^{2\alpha} & (e^{3\alpha}-1) \\ e^{\beta} & e^{2\beta} & (e^{3\beta}-1) \\ e^{\gamma} & e^{2\gamma} & (e^{3\gamma}-1) \end{vmatrix}$ is equal to

a) -2 b) -1 c) 0 d) 1

8. A root of the equation $\begin{vmatrix} 3-x & -6 & 3 \\ -6 & 3-x & 3 \\ 3 & 3 & -6-x \end{vmatrix} = 0$, is

a) 6 b) 3 c) 0 d) None of these

9. The value of $\begin{vmatrix} \log_3 512 & \log_4 3 \\ \log_3 8 & \log_4 9 \end{vmatrix} \times \begin{vmatrix} \log_2 3 & \log_8 3 \\ \log_3 4 & \log_3 4 \end{vmatrix}$ is

a) 7 b) 10 c) 13 d) 17

10. The value of the determinant $\begin{vmatrix} a^2 & a & 1 \\ \cos(nx) & \cos(n+1)x & \cos(n+2)x \\ \sin(nx) & \sin(n+1)x & \sin(n+2)x \end{vmatrix}$ is independent of

a) n b) a c) x d) None of these

11. If $x \neq 0$, $\begin{vmatrix} x+1 & 2x+1 & 3x+1 \\ 2x & 4x+3 & 6x+3 \\ 4x+4 & 6x+4 & 8x+4 \end{vmatrix} = 0$, then $x+1$ is equal to

a) x b) 0 c) $2x$ d) $3x$

12. The value of the determinant $\begin{vmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix}$ is equal to

a) -4 b) 0 c) 1 d) 4

13. If x, y, z are different from zero and

$\Delta \begin{vmatrix} a & b-y & c-z \\ a-x & b & c-z \\ a-x & b-y & c \end{vmatrix} = 0$, then the value of the expression $\frac{a}{x} + \frac{b}{y} + \frac{c}{z}$ is

a) 0 b) -1 c) 1 d) 2

14. If $a \neq p, b \neq q, c \neq r$ and $\begin{vmatrix} p & b & c \\ a & q & c \\ a & b & r \end{vmatrix} = 0$, then the value of $\frac{p}{p-a} + \frac{q}{q-b} + \frac{r}{r-c}$ is

a) 0 b) 1 c) -1 d) 2

15. The value of $\Delta = \begin{vmatrix} a & a+b & a+2b \\ a+2b & a & a+b \\ a+b & a+2b & a \end{vmatrix}$ is equal to

a) $9a^2(a+b)$ b) $9b^2(a+b)$ c) $a^2(a+b)$ d) $b^2(a+b)$

16. The value of θ lying between $\theta = 0$ and $\frac{\pi}{2}$ and satisfying

the equation $\begin{vmatrix} 1+\sin^2\theta & \cos^2\theta & 4\sin 4\theta \\ \sin^2\theta & 1+\cos^2\theta & 4\sin 4\theta \\ \sin^2\theta & \cos^2\theta & 1+4\sin 4\theta \end{vmatrix}$ is

a) $\frac{7\pi}{24}$ b) $\frac{5\pi}{24}$ c) $\frac{11\pi}{2}$ d) $\frac{\pi}{24}$

17. If $a_i^2 + b_i^2 + c_i^2 = 1 (i = 1, 2, 3)$ and $a_i a_j + b_i b_j + c_i c_j = 0 (i \neq j$ and $i, j = 1, 2, 3)$, then the value of $\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$ is

a) 0 b) $\frac{1}{2}$ c) 1 d) 2

18. If α, β, γ are the cube roots of 8, then $\begin{vmatrix} \alpha & \beta & \gamma \\ \beta & \gamma & \alpha \\ \gamma & \alpha & \beta \end{vmatrix} =$

a) 0 b) 1 c) 8 d) 2

19. If $\begin{vmatrix} 1+a & 1 & 1 \\ 1+b & 1+2b & 1 \\ 1+c & 1+c & 1+3c \end{vmatrix} = 0$, where $a \neq 0, b \neq 0, c \neq 0$, then $a^{-1} + b^{-1} + c^{-1}$ is

a) 4 b) -3 c) -2 d) -1

20. A root of the equation $\begin{vmatrix} 0 & x-a & x-b \\ x+a & 0 & x-c \\ x+b & x+c & 0 \end{vmatrix} = 0$, is

a) a b) b c) 0 d) 1

21. If $\begin{vmatrix} x & 2 & 3 \\ 2 & 3 & x \\ 3 & x & 2 \end{vmatrix} = \begin{vmatrix} 1 & x & 4 \\ x & 4 & 1 \\ 4 & 1 & x \end{vmatrix} = \begin{vmatrix} 0 & 5 & x \\ 5 & x & 0 \\ x & 0 & 5 \end{vmatrix} = 0$, then the value of x values $(x \in R)$:

a) 0 b) 5 c) -5 d) None of these

22. $\begin{vmatrix} bc & bc'+b'c & b'c' \\ ca & ca'+c'a & c'a' \\ ab & ab'+a'b & a'b' \end{vmatrix}$ is equal to

a) $(ab - a'b')(bc - b'c')(ca - c'a')$

b) $(ab + a'b')(bc + b'c')(ca + c'a')$

c) $(ab' - a'b)(bc' - b'c)(ca' - c'a)$

d) $(ab' + a'b)(bc' + b'c)(ca' + c'a)$

23. If a square matrix A is such that $AA^T = I = A^T A$ then $|A|$ is equal to

a) 0 b) ± 1 c) ± 2 d) None of these

24. If $\Delta_1=\begin{vmatrix}1 & 1 & 1\\ a & b & c\\ a^2 & b^2 & c^2\end{vmatrix}, \Delta_2=\begin{vmatrix}1 & bc & a\\ 1 & ca & b\\ 1 & ab & c\end{vmatrix}$, then

a) $\Delta_1+\Delta_2=0$ b) $\Delta_1+2\Delta_2=0$ c) $\Delta_1=\Delta_2$ d) $\Delta_1=2\Delta_2$

25. If $\begin{vmatrix}y+z & x & y\\ z+x & z & x\\ x+y & y & z\end{vmatrix}=k(x+y+z)(x-z)^2$, then k is equal to

a) $2xyz$ b) 1 c) xyz d) $x^2y^2z^2$

26. A is a square matrix of order 4 and I is a unit matrix, then it is true that

a) $\det(2A)=2\det(A)$ b) $\det(2A)=16\det(A)$

c) $\det(-A)=-\det(A)$ d) $\det(A+I)=det(A)+I$

27. If the matrix M_r is given by

$M_r=\begin{bmatrix}r & r-1\\ r-1 & r\end{bmatrix}$, $= 1,2,3,\ldots$, then the value of $\det(M_1)+\det(M_2)+\ldots+\det(M_{2008})$ is

a) 2007 b) 2008 c) $(2008)^2$ d) $(2007)^2$

28. l, m, n are the pth, qth and rth terms of an GP and all

Positive, then $\begin{vmatrix}\log l & p & 1\\ \log m & q & 1\\ \log n & r & 1\end{vmatrix}$ equals

a) 3 b) 2 c) 1 d) zero

29. The matrix $\begin{bmatrix}5 & 10 & 3\\ -2 & -4 & 6\\ -1 & -2 & b\end{bmatrix}$ is a singular matrix, if b is equal to

a) -3 b) 3 c) 0 d) For any value of b

30. Consider the system of equations

$a_1x+b_1y+c_1z=0$

$a_1x+b_2y+c_2z=0$

$a_3x+b_3y+c_3z=0$

If $\begin{vmatrix}a_1 & b_1 & c_1\\ a_2 & b_2 & c_2\\ a_3 & b_3 & c_3\end{vmatrix}=0$, then the system has

a) More than two solutions

b) One trivial and one non-trivial solutions

c) No solution

d) Only trivial solution (0,0,0)

31. If $\begin{vmatrix}x^n & x^{n+2} & x^{n+3}\\ y^n & y^{n+2} & y^{n+3}\\ z^n & z^{n+2} & z^{n+3}\end{vmatrix}=(y-z)(z-x)(x-y)$

$\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)$, then n is equal to

a) 2 b) -2 c) -1 d) 1

32. The determinant $\begin{vmatrix}a & b & a\alpha+b\\ b & c & b\alpha+c\\ a\alpha+b & b\alpha+c & 0\end{vmatrix}$ is equal to zero for all values of α, if

a) a, b, c are in AP b) a, b, c are in GP c) a, b, c are in HP d) None of these

33. The system of equations

$kx+y+z=1$

$x+ky+z=k$

$x+y+kz=k^2$

have no solution, if k equals

a) 0 b) 1 c) -1 d) -2

34. $\Delta=\begin{vmatrix}a & a+b & a+b+c\\ 3a & 4a+3b & 5a+4b+3c\\ 6a & 9a+6b & 11a+9b+6c\end{vmatrix}$, where $a=i, b=\omega, c=\omega^2$, then Δ is equal to

a) i b) $-\omega^2$ c) ω d) $-i$

35. If $\begin{vmatrix} a+b & b+c & c+a \\ b+c & c+a & a+b \\ c+a & a+b & b+c \end{vmatrix} = k \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$, then k is equal to

a) 4 b) 3 c) 2 d) 1

36. $\begin{vmatrix} \alpha & -\beta & 0 \\ 0 & \alpha & \beta \\ \beta & 0 & \alpha \end{vmatrix} = 0$, then

a) $\frac{\alpha}{\beta}$ is one of the cube roots of unity b) α is one of the cube roots of unity

c) β is one of the cube roots of unity d) $\alpha\beta$ is one of the cube roots of unity

37. $\Delta = \begin{vmatrix} 1/a & 1 & bc \\ 1/b & 1 & ca \\ 1/c & 1 & ab \end{vmatrix} =$

a) 0 b) abc c) $\frac{1}{abc}$ d) None of these

38. Using the factor theorem it is found that $a+b, b+c$ and $c+a$ are three factors of the determinant $\begin{vmatrix} -2a & a+b & a+c \\ b+a & -2b & b+c \\ c+a & c+b & -2c \end{vmatrix}$
The other factor in the value of the determinant is

a) 4 b) 2 c) $a+b+c$ d) None of these

39. The arbitrary constant on which the value of the determinant $\begin{vmatrix} 1 & \alpha & \alpha^2 \\ \cos(p-d)a & \cos pa & \cos(p-d)a \\ \sin(p-d)a & \sin pa & \sin(p-d)a \end{vmatrix}$ does not depend, is

a) α b) p c) d d) a

40. If $\omega$is imaginary root of unity, then the value of
$\begin{vmatrix} a & b\omega^2 & a\omega \\ b\omega & c & b\omega^2 \\ c\omega^2 & a\omega & c \end{vmatrix}$ is

a) $a^3+b^3+c^3$ b) a^2b-b^2c c) 0 d) $a^3+b^3+c^3-3abc$

41. If $\Delta_1 = \begin{vmatrix} 7 & x & 2 \\ -5 & x+1 & 3 \\ 4 & x & 7 \end{vmatrix}$ and $\Delta_2 = \begin{vmatrix} x & 2 & 7 \\ x+1 & 3 & -5 \\ x & 7 & 4 \end{vmatrix}$, then the value of x for which $\Delta_1 + \Delta_2 = 0$, is

a) 2 b) 0 c) Any real number d) None of these

42. If $\Delta_1 = \begin{vmatrix} x & b & b \\ a & x & b \\ a & a & x \end{vmatrix}$ and $\Delta_2 = \begin{vmatrix} x & b \\ a & x \end{vmatrix}$ are the given determinants, then

a) $\Delta_1 = 3(\Delta_2)^2$ b) $\frac{d}{dx}(\Delta_1) = 3\Delta_2$ c) $\frac{d}{dx}(\Delta_1) = 2\Delta_2$ d) $\Delta_1 = 3\Delta_2^{3/2}$

43. If $f(\theta) = \begin{vmatrix} \cos^2\theta & \cos\theta\sin\theta & -\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta & \cos\theta \\ \sin\theta & -\cos\theta & 0 \end{vmatrix}$. Then, for all θ

a) $f(\theta) = 0$ b) $f(\theta) = 1$ c) $f(\theta) = -1$ d) None of these

44. If $C = 2\cos\theta$, then the value of the determinant $\Delta = \begin{vmatrix} C & 1 & 0 \\ 1 & C & 1 \\ 6 & 1 & C \end{vmatrix}$ is

a) $\frac{\sin 4\theta}{\sin\theta}$ b) $\frac{2\sin^2 2\theta}{\sin\theta}$ c) $4\cos^2\theta(2\cos\theta - 1)$ d) None of these

45. If $f(x) = \begin{vmatrix} \sin x & \cos x & \tan x \\ x^3 & x^2 & x \\ 2x & 1 & 1 \end{vmatrix}$, then $\lim_{n\to 0} \frac{f(x)}{x^2}$, is

a) 3 b) -1 c) 0 d) 1

46. Let $[x]$ represent the greatest integer less than or equal to x, then the value of the determinant

$\begin{vmatrix} [e] & [\pi] & [\pi^2-6] \\ [\pi] & [\pi^2-6] & [e] \\ [\pi^2-6] & [e] & [\pi] \end{vmatrix}$ is

a) -8 b) 8 c) 10 d) None of these

47. If $A = \begin{bmatrix} 3 & 5 \\ 2 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 17 \\ 0 & -10 \end{bmatrix}$, then $|AB|$ is equal to

a) 80 b) 100 c) -110 d) 92

48. If $\Delta = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ 3 & 9 & 13 \end{vmatrix}$ and $\Delta' = \begin{vmatrix} 7 & 20 & 29 \\ 2 & 5 & 7 \\ 3 & 9 & 13 \end{vmatrix}$, then

a) $\Delta' = 3\Delta$ b) $\Delta' = \frac{3}{\Delta}$ c) $\Delta' = \Delta$ d) $\Delta' = 2\Delta$

49. $\begin{vmatrix} 2xy & x^2 & y^2 \\ x^2 & y^2 & 2xy \\ y^2 & 2xy & x^2 \end{vmatrix}$ is equal to

a) $(x^3+y^3)^2$ b) $(x^2+y^2)^3$ c) $-(x^2+y^2)^3$ d) $-(x^3+y^3)^2$

50. In a $\Delta\, ABC, a, b, c$ are sides and A, B, C are angles opposite to them, then the value of the determinant

$\begin{vmatrix} a^2 & b\sin A & c\sin A \\ b\sin A & 1 & \cos A \\ c\sin A & \cos A & 1 \end{vmatrix}$, is

a) 0 b) 1 c) 2 d) 3

51. $\begin{vmatrix} b^2c^2 & bc & b+c \\ c^2a^2 & ca & c+a \\ a^2b^2 & ab & a+b \end{vmatrix}$ is equal to

a) $\frac{1}{abc}(ab+bc+ca)$ b) $ab+bc+ca$ c) 0 d) $a+b+c$

52. If $a^{-1}+b^{-1}+c^{-1}=0$ such that $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = \lambda$ then value of λ is

a) 0 b) abc c) $-abc$ d) None of these

53. If $a, b, c,$ are in A.P., then the value of

$\begin{vmatrix} x+1 & x+2 & x+a \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix}$, is

a) 3 b) -3 c) 0 d) None of these

54. $\begin{vmatrix} a-b & b-c & c-a \\ x-y & y-z & z-x \\ p-q & p-r & r-p \end{vmatrix}$ is equal to

a) $a(x+y+z)+b(p+q+r)+c$ b) 0

c) $abc+xyz+ppr$ d) None of the above

55. $\begin{vmatrix} a-b+c & -a-b+c & 1 \\ a+b+2c & -a+b+2c & 2 \\ 3c & 3c & 3 \end{vmatrix}$ is

a) $6ab$ b) ab c) $12ab$ d) $2ab$

56. In the determinant $\begin{vmatrix} 0 & 1 & -2 \\ -1 & 0 & 3 \\ 2 & -3 & 0 \end{vmatrix}$, the value of cofactor to its minor of the element -3 is

a) -1 b) 0 c) 1 d) 2

57. If ω is a cube root of unity, then for polynomial is

$\begin{vmatrix} x+1 & \omega & \omega^2 \\ \omega & x+\omega^2 & 1 \\ \omega^2 & 1 & x+\omega \end{vmatrix}$

a) 1 b) ω c) ω^2 d) 0

58. If $\begin{vmatrix} x+a & b & c \\ a & x+b & c \\ a & b & x+c \end{vmatrix} = 0$, then x equals

a) $a+b+c$ b) $-(a+b+c)$ c) $0, a+b+c$ d) $0, -(a+b+c)$

59. If a, b, c are the sides of a ΔABC and A, B, C are respectively the angles opposite to them, then

$\begin{vmatrix} a^2 & b\sin A & c\sin A \\ b\sin A & 1 & \cos(B-C) \\ c\sin A & \cos(B-C) & 1 \end{vmatrix}$ equals

a) $\sin A - \sin B \sin C$ b) abc c) 1 d) 0

60. If $D_r = \begin{vmatrix} 2^{r-1} & 3^{r-1} & 4^{r-1} \\ x & y & z \\ 2^n - 1 & (3^n-1)/2 & (4^n-1)/3 \end{vmatrix}$, then the value of $\sum_{r=1}^{n} D_r$ is equal to

a) 1 b) -1 c) 0 d) None of these

61. If A, B and C are the angles of a triangle and

$\begin{vmatrix} 1 & 1 & 1 \\ 1+\sin A & 1+\sin B & \sin C \\ \sin A + \sin^2 A & \sin B + \sin^2 B & \sin C + \sin^2 C \end{vmatrix} = 0$

then the triangle must be

a) Equilateral b) Isosceles c) Any triangle d) Right angled

62. Let $A = \begin{bmatrix} 1 & \sin\theta & 1 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{bmatrix}$, where $0 \le \theta < 2\pi$. Then, which of the following is not correct?

a) Det $(A) = 0$ b) Det $(A) \in (-\infty, 0)$ c) Det $(A) \in [2,4]$ d) Det $(A) \in [-2, \infty)$

63. $\begin{vmatrix} 1 & 5 & \pi \\ \log_e e & 5 & \sqrt{5} \\ \log_{10} 10 & 5 & e \end{vmatrix}$ is equal to

a) $\sqrt{\pi}$ b) e c) 1 d) 0

64. If $a^2+b^2+c^2 = -2$ and

$f(x) = \begin{vmatrix} 1+a^2x & (1+b^2)x & (1+c^2)x \\ (1+a^2)x & (1+b^2x) & (1+c^2)x \\ (1+a^2)x & (1+b^2)x & (1+c^2x) \end{vmatrix}$, then $f(x)$ is a polynomial of degree

a) 2 b) 3 c) 0 d) 1

65. If $c < 1$ and the system of equations $x+y-1=0, 2x-y-c=0$ and $-bx+3by-c=0$ is consistent, then the possible real values of b are

a) $b \in \left(-3, \frac{3}{4}\right)$ b) $b \in \left(-\frac{3}{4}, 4\right)$ c) $b \in \left(-\frac{3}{4}, 3\right)$ d) None of these

66. The value of $\begin{vmatrix} 1 & 1 & 1 \\ (2^x+2^{-x})^2 & (3^x+3^{-x})^2 & (5^x+5^{-x})^2 \\ (2^x-2^{-x})^2 & (3^x-3^{-x})^2 & (5^x-5^{-x})^2 \end{vmatrix}$ is

a) 0 b) 30^x c) 30^{-x} d) 1

67. If A is an invertible matrix, then $\det(A^{-1})$ is equal to

a) $\det b(A)$ b) $\frac{1}{\det(A)}$ c) 1 d) None of these

68. If $a \neq 0, b \neq 0, c \neq 0$, then $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix}$ is equal to

a) abc b) $abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$ c) 0 d) $1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}$

69. If $f(x) = \begin{vmatrix} a & -1 & 0 \\ ax & a & -1 \\ ax^2 & ax & a \end{vmatrix}$, then $f(2x)-f(x)$ is equal to

a) ax b) $ax(2a+3x)$ c) $ax(2+3x)$ d) None of these

70. If $\begin{vmatrix} -12 & 0 & \lambda \\ 0 & 2 & -1 \\ 2 & 1 & 15 \end{vmatrix} = -360$, then the value of λ is

a) -1 b) -2 c) -3 d) 4

71. If ω is a complex cube root of unity, then

$\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix}$ is equal to

a) -1 b) 1 c) 0 d) ω

72. The value of $\begin{vmatrix} {}^{10}C_4 & {}^{10}C_5 & {}^{11}C_m \\ {}^{11}C_6 & {}^{11}C_7 & {}^{12}C_{m+2} \\ {}^{12}C_8 & {}^{12}C_9 & {}^{13}C_{m+4} \end{vmatrix} = 0$, when m is equal to

a) 6 b) 5 c) 4 d) 1

73. If $\begin{vmatrix} 1 & 1 & 0 \\ 2 & 0 & 3 \\ 5 & -6 & x \end{vmatrix} = 29$, then x is

a) 1 b) 2 c) 3 d) 4

74. $\begin{vmatrix} \sin^2 x & \cos^2 x & 1 \\ \cos^2 x & \sin^2 x & 1 \\ -10 & 12 & 2 \end{vmatrix} =$

a) 0

b) $12\cos^2 x - 10\sin^2 x$

c) $12\sin^2 x - 10\cos^2 x - 2$

d) $10\sin 2x$

75. If A and B are square matrices of order 3 such that $|A| = -1, |B| = 3$ then $|3AB|$ is equal to

a) -9 b) -81 c) -27 d) 81

76. If a, b, c are non-zero real numbers, then the system of equations

$(\alpha + a)x + \alpha y + \alpha z = 0$

$\alpha x + (\alpha + b)y + \alpha z = 0$

$\alpha x + \alpha y + (\alpha + c)z = 0$

has a non-trivial solution, if

a) $\alpha^{-1} = -(a^{-1} + b^{-1} + c^{-1})$

b) $\alpha^{-1} = a + b + c$

c) $\alpha + a + b + c = 1$

d) None of these

77. The determinant $\begin{vmatrix} a & b & a\alpha - b \\ b & c & b\alpha - c \\ 2 & 1 & 0 \end{vmatrix}$ vanishes, if

a) a, b, c are in AP b) $\alpha = \frac{1}{2}$ c) a, b, c are in GP d) Both (b) or (c)

78. If -9 is a root of the equation $\begin{vmatrix} x & 3 & 7 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix} = 0$, then the other two roots are

a) 2, 7 b) $-2, 7$ c) $2, -7$ d) $-2, -7$

79. If $ab + bc + ca = 0$ and $\begin{vmatrix} a - x & c & b \\ c & b - x & a \\ b & a & c - x \end{vmatrix} = 0$, then one of the value of x is

a) $(a^2 + b^2 + c^2)^{1/2}$

b) $\left[\frac{3}{2}(a^2 + b^2 + c^2)\right]^{1/2}$

c) $\left[\frac{1}{2}(a^2 + b^2 + c^2)\right]^{1/2}$

d) None of these

80. The roots of the equation $\begin{vmatrix} x - 1 & 1 & 1 \\ 1 & x - 1 & 1 \\ 1 & 1 & x - 1 \end{vmatrix} = 0$, are

a) 1, 2 b) −1, 2 c) 1, −2 d) −1, −2

81. $\begin{vmatrix} 1 & 2 & 3 \\ 1^3 & 2^3 & 3^3 \\ 1^5 & 2^5 & 3^5 \end{vmatrix}$ is equal to

a) 1! 213 b) 1! 3! 5! c) 6! d) 9!

82. If $A = \begin{bmatrix} \alpha & 2 \\ 2 & \alpha \end{bmatrix}$ and $|A^3| = 125$, then the value of $\alpha$is

a) ± 1 b) ± 2 c) ± 3 d) ± 5

83. The value of $\begin{vmatrix} x & 4 & y+z \\ y & 4 & z+x \\ z & 4 & x+y \end{vmatrix}$, is

a) 4 b) $x + y + z$ c) xyz d) 0

84. If A, B, C be the angles of a triangle, then $\begin{vmatrix} -1 & \cos C & \cos B \\ \cos C & -1 & \cos A \\ \cos B & \cos A & -1 \end{vmatrix}$ is equal to

a) 1 b) 0 c) $\cos A \cos B \cos C$ d) $\cos A + \cos B \cos C$

85. One factor of $\begin{vmatrix} a^2 + x & ab & ac \\ ab & b^2 + x & cb \\ ca & cb & c^2 + x \end{vmatrix}$ is

a) x^2

b) $(a^2 + x)(b^2 + x)(c^2 + x)$

c) $\frac{1}{x}$

d) None of these

86. If $\begin{vmatrix} x+1 & x+2 & x+3 \\ x+2 & x+3 & x+4 \\ x+a & x+b & x+c \end{vmatrix} = 0$ then a, b, c are in

a) AP b) HP c) GP d) None of these

87. If$A = \begin{vmatrix} 1 & 0 & 0 \\ x & 1 & 0 \\ x & x & 1 \end{vmatrix}$and $I = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$, then

$A^3 - 4A^2 + 3A + I$ is equal to

a) $3I$ b) I c) $-I$ d) $-2I$

88. Determinant $\begin{vmatrix} 1 & x & y \\ 2 & \sin x + 2x & \sin y + 3y \\ 3 & \cos x + 3x & \cos y + 3y \end{vmatrix}$ is equal to

a) $\sin(x - y)$ b) $\cos(x - y)$ c) $\cos(x + y)$ d) $xy(\sin(x - y)$

89. If a, b, c are the positive integers, then the determinant $\Delta = \begin{vmatrix} a^2 + x & ab & ac \\ ab & b^2 + x & bc \\ ac & bc & c^2 + x \end{vmatrix}$ is divisible by

a) x^3 b) x^2 c) $(a^2 + b^2 + c^2)$ d) None of these

90. If a, b, c are non-zero real numbers, then $\begin{vmatrix} bc & ca & ab \\ ca & ab & bc \\ ab & bc & ca \end{vmatrix}$ vanishes, when

a) $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 0$ b) $\frac{1}{a} - \frac{1}{b} - \frac{1}{c} = 0$ c) $\frac{1}{b} + \frac{1}{c} - \frac{1}{a} = 0$ d) $\frac{1}{b} - \frac{1}{c} - \frac{1}{a} = 0$

91. If $f(x) = \begin{vmatrix} 1 & 2(x-1) & 3(x-1)(x-2) \\ x-1 & (x-1)(x-2) & (x-1)(x-2)(x-3) \\ x & x(x-1) & x(x-1)(x-2) \end{vmatrix}$

Then, the value of $f(49)$ is

a) $49x$ b) $-49x$ c) 0 d) 1

92. if $\begin{vmatrix} 1+ax & 1+bx & 1+cx \\ 1+a_1x & 1+b_1x & 1+c_1x \\ 1+a_2x & 1+b_2x & 1+c_2x \end{vmatrix} = A_0 + A_1x + A_2x^2 + A_3x^3$, then A_0 is equal to

a) abc b) 0 c) 1 d) None of these

93. If A, B, C are the angles of a triangle, then the value of

$$\Delta = \begin{vmatrix} -1 & \cos C & \cos B \\ \cos C & -1 & \cos A \\ \cos B & \cos A & -1 \end{vmatrix} \text{ is}$$

a) $\cos A \cos B \cos C$ b) $\sin A \sin B \sin C$ c) 0 d) None of these

94. The value of the determinant

$$\begin{vmatrix} 1 & \cos(\beta-\alpha) & \cos(\gamma-\alpha) \\ \cos(\alpha-\beta) & 1 & \cos(\gamma-\beta) \\ \cos(\alpha-\gamma) & \cos(\beta-\gamma) & 1 \end{vmatrix} \text{ is}$$

a) $4\cos\alpha\cos\beta\cos\gamma$ b) $2\cos\alpha\cos\beta\cos\gamma$ c) $4\sin\alpha\sin\beta\sin\gamma$ d) None of these

95. If one root of determinant $\begin{vmatrix} x & 3 & 7 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix} = 0$, is$-9$, then the other two roots are

a) 2,7 b) 2,-7 c) -2,7 d) $-2, -7$

96. If $0 \le [x] < 2, -1 \le [y] < 1$ and $1 \le [z] < 3, [\cdot]$ denotes the greatest integer function, then the maximum value of the determinant

$$\Delta = \begin{vmatrix} [x]+1 & [y] & [z] \\ [x] & [y]+1 & [z] \\ [x] & [y] & [z]+1 \end{vmatrix}, \text{ is}$$

a) 2 b) 6 c) 4 d) None of these

97. If $D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+y \end{vmatrix}$ for $x \ne 0, y \ne 0$, then D is

a) Divisible by neither x nor y b) Divisible by both x and y

c) Divisible by x but not y d) Divisible by y but not x

98. If $f(x) = \begin{vmatrix} 1 & x & (x+1) \\ 2x & x(x-1) & x(x+1) \\ 3x(x-1) & x(x-1)(x-2) & x(x-1)(x+1) \end{vmatrix}$ then $f\,(11)$ equals

a) 0 b) 11 c) -11 d) 1

99. The roots of the equation $\begin{vmatrix} 1 & 4 & 20 \\ 1 & -2 & 5 \\ 1 & 2x & 5x^2 \end{vmatrix} = 0$

a) $-1, -2$ b) $-1, 2$ c) $1, -2$ d) 1, 2

100. One root of the equation

$$\begin{vmatrix} 3x-8 & 3 & 3 \\ 3 & 3x-8 & 3 \\ 3 & 3 & 3x-8 \end{vmatrix} =$$

a) 8/3 b) 2/3 c) 1/3 d) 16/3

101. If $\begin{vmatrix} \alpha & x & x & x \\ x & \beta & x & x \\ x & x & \gamma & x \\ x & x & x & \delta \end{vmatrix} = f(x) - xf'(x)$ then $f(x)$ is equal to

a) $(x-\alpha)(x-\beta)(x-\gamma)(x-\delta)$

b) $(x+\alpha)(x+\beta)(x+\gamma)(x+\delta)$

c) $2(x-\alpha)(x-\beta)(x-\gamma)(x-\delta)$

d) None of these

102. In ΔABC if $\begin{vmatrix} 1 & a & b \\ 1 & c & a \\ 1 & b & c \end{vmatrix} = 0$,then

$\sin^2 A + \sin^2 B + \sin^2 C$ is equal to

a) $\frac{4}{9}$ b) $\frac{9}{4}$ c) $3\sqrt{3}$ d) 1

103. The value of determinant $\begin{vmatrix} b+c & a+b & a \\ c+a & b+c & b \\ a+b & c+a & c \end{vmatrix}$ is equal to

a) $a^3+b^3+c^3-3\,abc$ b) $2abc(a+b+c)$ c) 0 d) None of these

104. If $n=3k$ and $1\;\omega,\omega^2$ are the cube roots of unity, then $\Delta=\begin{vmatrix} 1 & \omega^n & \omega^{2n} \\ \omega^{2n} & 1 & \omega^n \\ \omega^n & \omega^{2n} & 1 \end{vmatrix}$ has the value

a) 0 b) ω c) ω^2 d) 1

105. If $\begin{vmatrix} x & 3 & 6 \\ 3 & 6 & x \\ 6 & x & 3 \end{vmatrix}=\begin{vmatrix} 2 & x & 7 \\ x & 7 & 2 \\ 7 & 2 & x \end{vmatrix}=\begin{bmatrix} 4 & 5 & x \\ 5 & x & 4 \\ x & 4 & 5 \end{bmatrix}=0$,then x is equal to

a) 9 b) -9 c) 0 d) -1

106. the system of simultaneous equations

$kx+2y-z=1$

$(k-1)y-2z=2$

$(k+2)z=3$

Have a unique solution if k equals

a) -2 b) -1 c) 0 d) 1

107. If α,β are non-real numbers satisfying $x^3-1=0$, then the value of $\begin{vmatrix} \lambda+1 & \alpha & \beta \\ \alpha & \lambda+\beta & 1 \\ \beta & 1 & \lambda+\alpha \end{vmatrix}$ is equal to

a) 0 b) λ^3 c) λ^3+1 d) λ^3-1

108. If x,y,z are different from zero and $\Delta=\begin{vmatrix} a & b-y & c-z \\ a-x & b & c-z \\ a-x & b-y & c \end{vmatrix}=0$, then the value of expression $\frac{a}{x}+\frac{b}{y}+\frac{c}{z}$ is

a) 0 b) -1 c) 1 d) 2

109. The value of the determinant

$\begin{vmatrix} 1 & \cos(\alpha-\beta) & \cos\alpha \\ \cos(\alpha-\beta) & 1 & \cos\beta \\ \cos\alpha & \cos\beta & 1 \end{vmatrix}$ is

a) 0 b) 1 c) $\alpha^2-\beta^2$ d) $\alpha^2+\beta^2$

110. If A,b,C are the angles of a triangle, then the determinant

$\Delta=\begin{vmatrix} \sin 2A & \sin C & \sin B \\ \sin C & \sin 2B & \sin A \\ \sin B & \sin A & \sin 2C \end{vmatrix}$ is equal to

a) 1 b) -1 c) $\sin A+\sin B+\sin C$ d) None of these

111. $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$ is equal to

a) 0 b) $a+b+c$ c) $(a+b+c)^2$ d) $(a+b+c)^3$

112. A and B are two non-zero square matrices such that $AB=O$. Then,

a) Both A and B are singular

b) Either of them is singular

c) Neither matrix is singular

d) None of these

113. The system of linear equations

$x+y+z=2$

$2\,x+y-z=3$

$3\,x+2\,y+kz=4$

Has a unique solution, is

a) $k\neq 0$ b) $-1<k<1$ c) $-2<k<2$ d) $k=0$

114. If $a_1,a_2,\dots\dots,a_n,\dots\dots$are in GP and $a_i>0$ for each i, then the determinant

$\Delta=\begin{vmatrix}\log a_n & \log a_{n+2} & \log a_{n+4}\\ \log a_{n+6} & \log a_{n+8} & \log a_{n+10}\\ \log a_{n+12} & \log a_{n+14} & \log a_{n+16}\end{vmatrix}$ is equal to

a) 0 b) 1 c) 2 d) n

115. The value of $\begin{vmatrix}11 & 12 & 13\\ 12 & 13 & 14\\ 13 & 14 & 15\end{vmatrix}$, is

a) 1 b) 0 c) -1 d) 67

116. The determinant $\begin{vmatrix}\cos C & \tan A & 0\\ \sin B & 0 & -\tan A\\ 0 & \sin B & \cos C\end{vmatrix}$ has the value, where A, B, C are angles of a triangle

a) 0 b) 1 c) $\sin A \sin B$ d) $\cos A \cos B \cos C$

117. If $0 < \theta < \pi$ and the system of equations

$(\sin\theta)x + y + z = 0$

$x + (\cos\theta)y + z = 0$

$(\sin\theta)x + (\cos\theta)y + z = 0$

Has a non-trivial solution, then $\theta =$

a) $\frac{\pi}{6}$ b) $\frac{\pi}{4}$ c) $\frac{\pi}{3}$ d) $\frac{\pi}{2}$

118. Let $\omega = -\frac{1}{2} + i\frac{\sqrt{3}}{2}$, then the value of the determinant

$\begin{vmatrix}1 & 1 & 1\\ 1 & -1-\omega^2 & \omega^2\\ 1 & \omega^2 & \omega^4\end{vmatrix}$, is

a) 3ω b) $3\omega(\omega-1)$ c) $3\omega^2$ d) $3\omega(1-\omega)$

119. Let $ax^6 + bx^5 + cx^4 + dx^3 + ex^2 + fx + g = \begin{vmatrix}(x+1) & (x^2+2) & (x^2+x)\\ (x^2+x) & (x^2+1) & (x^2+2)\\ (x^2+2) & (x^2+x) & (x+1)\end{vmatrix}$. Then,

a) $f = 3, g = -5$ b) $f = -3, g = -5$ c) $f = -3, g = -9$ d) None of these

120. In a ΔABC ,if $\begin{vmatrix}1 & a & b\\ 1 & c & a\\ 1 & b & c\end{vmatrix} = 0$, then $\sin^2 A + \sin^2 B + \sin^2 C$ is equal to

a) $\frac{9}{4}$ b) $\frac{4}{9}$ c) 1 d) $3\sqrt{3}$

121. The value of the determinant $\begin{vmatrix}1 & 1 & 1 & 1\\ 1 & 2 & 3 & 4\\ 1 & 3 & 6 & 10\\ 1 & 4 & 10 & 20\end{vmatrix}$ is equal to

a) 0 b) -1 c) 1 d) 10

122. If $\Delta(x) = \begin{vmatrix}f(x)+f(-x) & 0 & x^4\\ 3 & f(x)-f(-x) & \cos x\\ x^4 & 2x & f(x)f(-x)\end{vmatrix}$ (where $f(x)$ is a real valued function of x), then the value of $\int_{-2}^{2} x^4\Delta(x)$

a) Depends upon the function $f(x)$ b) is 4

c) is -4 d) is zero

123. The value of $\begin{vmatrix}\cos(x-a) & \cos(x+a) & \cos x\\ \sin(x+a) & \sin(x-a) & \sin x\\ \cos a \tan x & \cos a \cot x & \operatorname{cosec} 2x\end{vmatrix}$ is equal

a) 1 b) $\sin a \cos a$ c) 0 d) $\sin x \cos x$

124. The roots of the equation

$$\begin{vmatrix} 3x^2 & x^2+x\cos\theta+\cos^2\theta & x^2+x\sin\theta+\sin^2\theta \\ x^2+x\cos\theta+\cos^2\theta & 3\cos^2\theta & 1+\frac{\sin 2\theta}{2} \\ x^2+x\sin\theta+\sin^2\theta & 1+\frac{\sin 2\theta}{2} & 3\sin^2\theta \end{vmatrix}=0$$

a) $\sin\theta, \cos\theta$ b) $\sin^2\theta, \cos^2\theta$ c) $\sin\theta, \cos^2\theta$ d) $\sin^2\theta, \cos\theta$

125. If A is a square matrix of order n such that its elements are polynomial in x and its r-rows become identical for $x=k$, then

a) $(x-k)^r$ is a factor of $|A|$

b) $(x-k)^{r-1}$ is a factor of $|A|$

c) $(x-k)^{r+1}$ is a factor $|A|$

d) $(x-k)^r$ is a factor of A

126. If $\begin{vmatrix} x^2+x & 3x-1 & -x+3 \\ 2x+1 & 2+x^2 & x^3-3 \\ x-3 & x^2+4 & 3x \end{vmatrix}$
$=a_0+a_1x+a_2x^2+_{,}\ldots+a_7x^7,$
The value of $a_0$is

a) 25 b) 24 c) 23 d) 21

127. If $\begin{vmatrix} a & \cot\frac{A}{2} & \lambda \\ b & \cot\frac{B}{2} & \mu \\ c & \cot\frac{C}{2} & \gamma \end{vmatrix}=0$ where, a, b, c A, B and C are elements of a ΔABC with usual meaning. Then, the value of $a(\mu-\gamma)+b(\gamma-\lambda)+c(\lambda-\mu)$ is

a) 0 b) abc c) $ab+bc+ca$ d) $2\,abc$

128. The value of the determinant $\begin{vmatrix} bc & ca & ab \\ p & q & r \\ 1 & 1 & 1 \end{vmatrix}$, where a, b, c are the p^{th}, q^{th} and r^{th} terms of a H.P., is

a) $p+q+r$ b) $(a+b+c)$ c) 1 d) None of these

129. If $a, b. c$ are in AP, then the value of $\begin{vmatrix} x+2 & x+3 & x+a \\ x+4 & x+5 & x+b \\ x+6 & x+7 & x+c \end{vmatrix}$ is

a) $x-(a+b+c)$ b) $9x^2+a+b+c$ c) $a+b+c$ d) 0

130. For the values of A, B, C and P, Q, R the value of

$\begin{vmatrix} \cos(A-P) & \cos(A-Q) & \cos(A-R) \\ \cos(B-P) & \cos(B-Q) & \cos(B-R) \\ \cos(C-P) & \cos(C-Q) & \cos(C-R) \end{vmatrix}$ is

a) 0 b) $\cos A\cos B\cos C$ c) $\sin A\sin B\sin C$ d) $\cos P\cos Q\cos R$

131. If $\Delta(x)=\begin{vmatrix} x^n & \sin x & \cos x \\ n! & \sin\frac{n\pi}{2} & \cos\frac{n\pi}{2} \\ a & a^2 & a^3 \end{vmatrix}$, then the value of $\frac{d^n}{dx^n}[\Delta(x)]$at $x=0$ is

a) -1 b) 0 c) 1 d) Dependent of a

132. For positive numbers x, y and z, the numerical value of the determinant $\begin{vmatrix} 1 & \log_x y & \log_x z \\ \log_y x & 1 & \log_y z \\ \log_z x & \log_z y & 1 \end{vmatrix}$ is

a) 0 b) 1 c) $\log_e xyz$ d) None of these

133. The value of the determinant $\begin{vmatrix} 15! & 16! & 17! \\ 16! & 17! & 18! \\ 17! & 18! & 19! \end{vmatrix}$ is equal to

a) $15!+16!$ b) $2(15!)(16!)(17!)$ c) $15!+16!+17!$ d) $16!+17!$

134. If $\Delta=\begin{vmatrix} 3 & 4 & 5 & x \\ 4 & 5 & 6 & y \\ 5 & 6 & 7 & z \\ x & y & z & 0 \end{vmatrix}$, then Δ equals

a) $(y-2z+3x)^2$
b) $(x-2y+z)^2$
c) $(x+y+z)^2$
d) $x^2+y^2+z^2-xy-yz-zx$

135. If the system of equations $2x+3y+5=0, x+ky+5=0, kx-12y-14=0$ be consistent, then value of k is

a) $-2, \frac{12}{5}$ b) $-1, \frac{1}{5}$ c) $-6, \frac{17}{5}$ d) $6, -\frac{12}{5}$

136. If $\begin{vmatrix} b^2+c^2 & ab & ac \\ ab & c^2+a^2 & bc \\ ca & cb & a^2+b^2 \end{vmatrix} = k\, a^2\, b^2\, c^2$, then k is equal to

a) 3 b) 2 c) 4 d) None of these

137. The repeated factor of the determinant
$\begin{vmatrix} y+z & x & y \\ z+x & z & x \\ x+y & y & z \end{vmatrix}$, is

a) $z-x$ b) $x-y$ c) $y-z$ d) None of these

138. The determinant $\begin{vmatrix} 4+x^2 & -6 & -2 \\ -6 & 9+x^2 & 3 \\ -2 & 3 & 1+x^2 \end{vmatrix}$ is not divisible by

a) x b) x^3 c) $14+x^2$ d) x^5

139. If a, b, c are different, then the value of x satisfying $\begin{vmatrix} 0 & x^2-a & x^3-b \\ x^2+a & 0 & x^2+c \\ x^4+a & x-c & 0 \end{vmatrix} = 0$ is

a) a b) b c) c d) 0

140. Determinant $\begin{vmatrix} b^2+c^2 & a^2 & a^2 \\ b^2 & c^2+a^2 & b^2 \\ c^2 & c^2 & a^2+b^2 \end{vmatrix}$ is equal to

a) abc b) $4\,abc$ c) $4a^2b^2c^2$ d) $a^2b^2c^2$

141. If $a \neq p, b \neq q, c \neq r$ and $\begin{vmatrix} p & b & c \\ p+a & q+b & 2c \\ a & b & r \end{vmatrix} = 0$,then

$\frac{p}{p-a} + \frac{q}{q-b} + \frac{r}{r-c}$ is equal to

a) 0 b) 1 c) 2 d) 3

142. $\begin{vmatrix} a+b+2c & a & b \\ c & 2a+b+c & b \\ c & a & a+2b+c \end{vmatrix}$ is equal to

a) $(a+b+c)^2$ b) $2(a+b+c)^2$
c) $(a+b+c)^3$ d) $2(a+b+c)^3$

143. If [ ] denotes the greatest integer less than or equal to the real number under consideration and $-1 \leq x < 0; 0 \leq y < 1; 1 \leq z < 2$, then the value of the determinant $\begin{vmatrix} [x]+1 & [y] & [z] \\ [x] & [y]+1 & [z] \\ [x] & [y] & [z]+1 \end{vmatrix}$ is

a) $[x]$ b) $[y]$ c) $[z]$ d) None of these

144. The values of x for which the given matrix
$\begin{bmatrix} -x & x & 2 \\ 2 & x & -x \\ x & -2 & -x \end{bmatrix}$ will be non-singular, are

a) $-2 \leq x \leq 2$ b) For all x other then 2 and -2
c) $x \geq 2$ d) $x \leq -2$

145. If all the elements in a square matrix A of order 3 are equal to 1 or -1, then $|A|$, is

a) An odd number b) An even number c) An imaginary number d) A real number

146. Let a, b, c be such that $(b+c) \neq 0$ and

$$\begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix}$$
$$+\begin{vmatrix} a+1 & b+1 & c-1 \\ a-1 & b-1 & c+1 \\ (-1)^{n+2}a & (-1)^{n+1}b & (-1)^{n}c \end{vmatrix}^{1} = 0$$

Then the value of n is

a) Zero b) Any even integer c) Any odd integer d) Any integer

147. Determinant $\begin{vmatrix} 1/a & a^2 & bc \\ 1/b & b^2 & ca \\ 1/c & c^2 & ab \end{vmatrix}$ is equal to

a) abc b) $\frac{1}{abc}$ c) $ab+bc+ca$ d) 0

148. One root of the equation $\begin{vmatrix} x & -6 & -1 \\ 2 & -3x & x-3 \\ -3 & 2x & x+2 \end{vmatrix}$=0 is

a) 0 b) 1 c) -1 d) 3

149. The value of $\begin{vmatrix} a^2 & b^2 & c^2 \\ (a+1)^2 & (b+1)^2 & (c+1)^2 \\ (a-1)^2 & (b-1)^2 & (c-1)^2 \end{vmatrix}$ is

a) $4\begin{vmatrix} a^2 & b^2 & c^2 \\ a & b & c \\ 1 & 1 & 1 \end{vmatrix}$ b) $3\begin{vmatrix} a^2 & b^2 & c^2 \\ a & b & c \\ 1 & 1 & 1 \end{vmatrix}$ c) $2\begin{vmatrix} a^2 & b^2 & c^2 \\ a & b & c \\ 1 & 1 & 1 \end{vmatrix}$ d) None of these

150. The value of the determinant

$\Delta=\begin{vmatrix} \frac{1-a_1^3 b_1^3}{1-a_1 b_1} & \frac{1-a_1^3 b_2^3}{1-a_1 b_2} & \frac{1-a_1^3 b_3^3}{1-a_1 b_3} \\ \frac{1-a_2^3 b_1^3}{1-a_2 b_1} & \frac{1-a_2^3 b_2^3}{1-a_2 b_2} & \frac{1-a_2^3 b_3^3}{1-a_2 b_3} \\ \frac{1-a_3^3 b_1^3}{1-a_3 b_1} & \frac{1-a_3^3 b_2^3}{1-a_3 b_2} & \frac{1-a_3^3 b_3^3}{1-a_3 b_3} \end{vmatrix}$, is

a) 0
b) Dependent only on a_1, a_2, a_3
c) Dependent only on b_1, b_2, b_3
d) Dependent on $a_1, a_2, a_3\ b_1, b_2, b_3$

151. If $A=\begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}$, then the value of the determinant$|A^{2009}-5A^{2008}|$is

a) -6 b) -5 c) -4 d) 4

152. If $f(x)=\begin{vmatrix} x-3 & 2x^2-18 & 3x^3-81 \\ x-5 & 2x^2-50 & 4x^3-500 \\ 1 & 2 & 3 \end{vmatrix}$,then

$f(1).f(3)+f(3).f(5)+f(5).f(1)$ is equal to

a) $f(1)$ b) $f(3)$ c) $f(1)+f(3)$ d) $f(1)+f(5)$

153. The value of the determinant$\begin{vmatrix} x & a & b+c \\ x & b & c+a \\ x & c & a+b \end{vmatrix}=0$, if

a) $x=a$ b) $x=b$ c) $x=c$ d) x has any value

154. If the system of equations $x+ky-z=0, 3x-ky-z=0$ and $x-3y+z=0$ has non-zero solution then k is equal to

a) -1 b) 0 c) 1 d) 2

155. If $\begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix}=0$ and x, y, z are all distinct, then xyz is equal to

a) -1 b) 1 c) 0 d) 3

156. Let $[x]$ represent the greatest integer less than or equal to x, then the value of the determinant $\begin{vmatrix} [e] & [\pi] & [\pi^2-6] \\ [\pi] & [\pi^2-6] & [e] \\ [\pi^2-6] & [e] & [\pi] \end{vmatrix}$ is

a) -8 b) 8 c) 10 d) None of these

157. The determinant $\Delta = \begin{vmatrix} a & b & ax+b \\ b & c & bx+c \\ ax+b & bx+c & 0 \end{vmatrix}$ is equal to zero, if

a) a, b, c, are in A.P.

b) a, b, c, are in G.P.

c) a, b, c, are in H.P.

d) α is a root of $ax^2+bx+c=0$

158. Consider the following statements :

1. The determinants $\begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix}$ and $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$ are not identically equal.

2. For $a>0, b>0, c>0$ the value of the determinant $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ is always positive.

3. If $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \begin{vmatrix} a_1 & b_1 & 1 \\ a_2 & b_2 & 1 \\ a_3 & b_3 & 1 \end{vmatrix}$, then the two triangles with vertices $(x_1, y_1), (x_2, y_2), (x_3, y_3)$ and $(a_1, b_1), (a_2, b_2), (b_3, b_3)$ must be congruent. Which of the statement given above is/are correct?

a) Only (1) b) Only (2) c) Only (3) d) None of these

159. The arbitrary constant on which the value of the

Determinant $\begin{vmatrix} 1 & a & a^2 \\ \cos(p-d)\,a & \cos pa & \cos(p-d)\,a \\ \sin(p-d)\,a & \sin pa & \sin(p-d)\,a \end{vmatrix}$

Does not depend, is

a) α b) p c) d d) a

160. **If $\begin{vmatrix} a+x & a-x & a-x \\ a-x & a+x & a-x \\ a-x & a-x & a+x \end{vmatrix} = 0$,then x is equal to**

a) $0, 2a$ b) $a, 2a$ c) $0, 3a$ d) None of these

161. If the equations $2x+3y+1=0, 3x+y-2=0$ and $ax+2y-b=0$ are consistent, then

a) $a-b=2$ b) $a+b+1=0$ c) $a+b=3$ d) $a-b-8=0$

162. If $\Delta(x) = \begin{vmatrix} 1 & \cos x & 1-\cos x \\ 1+\sin x & \cos x & 1+\sin x-\cos x \\ \sin x & \sin x & 1 \end{vmatrix}$, then $\int_0^{\pi/2} \Delta(x)dx$ is equal to

a) $\frac{1}{4}$ b) $\frac{1}{2}$ c) 0 d) $-\frac{1}{2}$

163. If the system of equations

$x + a\,y + a\,z = 0$

$b\,x + y + b\,z = 0$

$c\,x + c\,y + z = 0$

Where a, b and c are non-zero non-unity, has a non-trivial solution, then the value of $\frac{a}{1-a} + \frac{b}{1-b} + \frac{c}{1-c}$ is

a) 0 b) 1 c) -1 d) $\frac{abc}{a^2+b^2+c^2}$

164. The system of equations $3x-2y+z=0, \lambda x-14y+15z=0, x+2y-3z=0$ has a solution other than $x=y=z=0$ then λ is equal to

a) 1 b) 2 c) 3 d) 5

165. Let $D_r = \begin{vmatrix} 2^{r-1} & 2.3^{r-1} & 4.5^{r-1} \\ \alpha & \beta & \gamma \\ 2^n - 1 & 3^n - 1 & 5^n - 1 \end{vmatrix}$. Then, the value of $\sum_{r=1}^{n} D_r$ is

a) $\alpha\,\beta\,\gamma$
b) $2^n\,\alpha + 2^n\,\beta + 4^n\,\gamma$
c) $2\,\alpha + 3\,\beta + 4\,\gamma$
d) None of these

166. In the interval $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$, the number of real solutions of the equations $\begin{vmatrix} \sin x & \cos x & \cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix} = 0$ is

a) 0
b) 2
c) 1
d) 3

167. If A, B and C are the angles of a triangle and

$\begin{vmatrix} 1 & 1 & 1 \\ 1 + \sin A & 1 + \sin B & 1 + \sin C \\ \sin A + \sin^2 A & \sin B + \sin^2 B & \sin C + \sin^2 C \end{vmatrix} = 0$ then the triangle ABC is

a) Isosceles
b) Equilateral
c) Right angled isosceles
d) None of these

168. If $A = \begin{vmatrix} a & b & c \\ x & y & z \\ p & q & r \end{vmatrix}$ and $B = \begin{vmatrix} q & -b & y \\ -p & a & -x \\ r & -c & z \end{vmatrix}$, then

a) $A = 2B$
b) $A = B$
c) $A = -B$
d) None of these

169. If $a = 1 + 2 + 4 + \cdots$ to n terms, $b = 1 + 3 + 9 + \cdots$ to n terms and $c = 1 + 5 + 25 + \cdots$ to n terms, then $\begin{vmatrix} a & 2b & 4c \\ 2 & 2 & 2 \\ 2^n & 3^n & 5^n \end{vmatrix}$ equals

a) $(30)^n$
b) $(10)^n$
c) 0
d) $2^n + 3^n + 5^n$

170. If $c = 2\cos\theta$, then the value of the determinant

$\Delta = \begin{vmatrix} c & 1 & 0 \\ 1 & c & 1 \\ 6 & 1 & c \end{vmatrix}$ is

a) $\dfrac{\sin 4\theta}{\sin\theta}$
b) $\dfrac{2\sin^2 2\theta}{\sin\theta}$
c) $4\cos^2\theta(2\cos\theta - 1)$
d) None of these

171. The value of $\Delta = \begin{vmatrix} 1^2 & 2^2 & 3^2 \\ 2^2 & 3^2 & 4^2 \\ 3^2 & 4^2 & 5^2 \end{vmatrix}$, is

a) 8
b) −8
c) 400
d) 1

172. The factors of $\begin{vmatrix} x & a & b \\ a & x & b \\ a & b & x \end{vmatrix}$ are

a) $x - a, x - b$, and $x + a + b$
b) $x + a, x + b$ and $x + a + b$
c) $x + a, x + b$ and $x - a - b$
d) $x - a, x - b$ and $x - a - b$

173. Coefficient of x in

$f(x) = \begin{vmatrix} x & (1 + \sin x)^2 & \cos x \\ 1 & \log(1 + x) & 2 \\ x^2 & (1 + x)^2 & 0 \end{vmatrix}$, is

a) 0
b) 1
c) −2
d) Cannot be determined

174. If $a \neq b, b, c$ satisfy $\begin{vmatrix} a & 2b & 2c \\ 3 & b & c \\ 4 & a & b \end{vmatrix} = 0$, then $abc =$

a) $a + b + c$
b) 0
c) b^3
d) $ab + bc$

175. Which one of the following is correct?

If A non-singular matrix, then

a) $\det(A^{-1}) = \det(A)$ b) $\det(A^{-1}) = \frac{1}{\det(A)}$ c) $\det(A^{-1}) = 1$ d) None of these

176. If $\begin{vmatrix} a & b & 0 \\ 0 & a & b \\ b & 0 & a \end{vmatrix} = 0$, then

a) a is one of the cube roots of unity b) b is one of the cube roots of unity

c) $\left(\frac{a}{b}\right)$ is one of the cube roots of unity d) $\left(\frac{a}{b}\right)$ is one of the cube roots of -1

177. If $\begin{vmatrix} b+c & c+a & a+b \\ a+b & b+c & c+a \\ c+a & a+b & b+c \end{vmatrix} = k \begin{vmatrix} a & b & c \\ c & a & b \\ d & c & a \end{vmatrix}$, then the value of k, is

a) 1 b) 2 c) 3 d) 4

178. If the value of the determinant $\begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix}$ is positive, then

a) $abc > 1$ b) $abc > -8$ c) $abc < -8$ d) $abc > -2$

179. The value of the determinant

$\begin{vmatrix} \cos\alpha & -\sin\alpha & 1 \\ \sin\alpha & \cos\alpha & 1 \\ \cos(\alpha+\beta) & -\sin(\alpha+\beta) & 1 \end{vmatrix}$ is

a) Independent of α b) Independent of β

c) Independent of α and β d) None of these

180. If B is a non-singular matrix and A is a square matrix such that $B^{-1}AB$ exists, then $\det(B^{-1}AB)$ is equal to

a) $\det(A^{-1})$ b) $\det(B^{-1})$ c) $\det(B)$ d) $\det(A)$

181. If matrix $\begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & 3 \\ \lambda & -3 & 0 \end{bmatrix}$ is singular, then λ is equal to

a) -2 b) -1 c) 1 d) 2

182. If x, y, z are in AP, then the value of the det A is, where

$A = \begin{vmatrix} 4 & 5 & 6 & x \\ 5 & 6 & 7 & y \\ 6 & 7 & 8 & z \\ x & y & z & 0 \end{vmatrix}$

a) 0 b) 1 c) 2 d) None of these

183. If $\Delta_r = \begin{vmatrix} 1 & n & n \\ 2r & n^2+n+1 & n^2+n \\ 2r-1 & n^2 & n^2+n+1 \end{vmatrix}$ and $\sum_{r=1}^{n} \Delta_r = 56$, then n equals

a) 4 b) 6 c) 7 d) 8

184. $\begin{vmatrix} 1 & a & a^2-bc \\ 1 & b & b^2-ac \\ 1 & c & c^2-ab \end{vmatrix}$ is equal to

a) 0 b) $a^3+b^3+c^3-3abc$

c) $3abc$ d) $(a+b+c)^3$

185. If the matrix M_r is given by $M_r = \begin{bmatrix} r & r-1 \\ r-1 & r \end{bmatrix}$ $r = 1, 2, 3 \ldots$, then the value of $\det(M_1) + \det(M_2) + \ldots + \det(M_{2008})$ is

a) 2007 b) 2008 c) $(2008)^2$ d) $(2007)^2$

186. If ω is the cube root of unity, then $\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix}$ is equal to

a) 1 b) 0 c) ω d) ω^2

187. If $1, \omega, \omega^2$ are the cube roots of unity, then

$\Delta = \begin{vmatrix} 1 & \omega^n & \omega^{2n} \\ \omega^n & \omega^{2n} & 1 \\ \omega^{2n} & 1 & \omega^n \end{vmatrix}$ is equal to

a) 0 b) 1 c) ω d) ω^2

188. The value of the following determinant is

$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix}$

a) $(a-b)(b-c)(c-a)(a+b+c)$
b) $abc(a+b)(b+c)(c+a)$
c) $(a-b)(b-c)(c-a)$
d) None of the above

189. The value of $\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$, is

a) $6\,abc$
b) $a+b+c$
c) $4\,abc$
d) abc

190. The value of $\begin{vmatrix} \log_5 729 & \log_3 5 \\ \log_5 27 & \log_9 25 \end{vmatrix} \begin{vmatrix} \log_3 5 & \log_{27} 5 \\ \log_5 9 & \log_5 9 \end{vmatrix}$ is equal to

a) 1
b) 6
c) $\log_5 9$
d) $\log_3 5 . \log_5 81$

191. If $a+b+c=0$, then the solution of the equation $\begin{vmatrix} a-x & c & b \\ c & b-x & a \\ b & a & c-x \end{vmatrix} = 0$ is

a) 0
b) $\pm\frac{3}{2}(a^2+b^2+c^2)$
c) $0, \pm\sqrt{\frac{3}{2}(a^2+b^2+c^2)}$
d) $0, \pm\sqrt{(a^2+b^2+c^2)}$

192. If a, b and c are all different from zero and $\Delta = \begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = 0$, then the value of $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}$ is

a) abc
b) $\frac{1}{abc}$
c) $-a-b-c$
d) -1

193. If $(\omega \neq 1)$ is a cubic root of unity, then

$\begin{vmatrix} 1 & 1+i+\omega^2 & \omega^2 \\ 1-i & -1 & \omega^2-1 \\ -i & -1+\omega-i & -1 \end{vmatrix}$

a) zero
b) 1
c) i
d) ω

194. The value of $\sum_{n=1}^{N} U_n$ if $U_n = \begin{vmatrix} n & 1 & 5 \\ n^2 & 2N+1 & 2N+1 \\ n^3 & 3N^2 & 3N \end{vmatrix}$, is

a) 0
b) 1
c) -1
d) None of these

195. The integer represented by the determinant

$\begin{bmatrix} 215 & 342 & 511 \\ 6 & 7 & 8 \\ 36 & 49 & 54 \end{bmatrix}$ is exactly divisible by

a) 146
b) 21
c) 20
d) 335

196. If A is a 3×3non-singular matrix, then det $(A^{-1}\text{adj } A)$ is equal to

a) det A
b) 1
c) $(\det A)^2$
d) $(\det A)^{-1}$

197. Let $A = \begin{vmatrix} 1 & \sin\theta & 1 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{vmatrix}$, where $0 \leq \theta \leq 2\pi$, then the range of o$|A|$ is

a) $(2, 4)$
b) $[2, 4]$
c) $[2, 4)$
d) All of these

198. In a third order determinant, each element of the first column consists of sum of two terms, each element of the second column consists of sum of three terms and each element of the third column consists of sum of four terms. Then, it can be decomposed into n determinant, where n has the value

a) 1
b) 9
c) 16
d) 24

199. If $a_1, a_2, \ldots, a_n, \ldots$, are in GP, then the determinant

$\Delta = \begin{vmatrix} \log a_n & \log a_{n+1} & \log a_{n+2} \\ \log a_{n+3} & \log a_{n+4} & \log a_{n+5} \\ \log a_{n+6} & \log a_{n+7} & \log a_{n+8} \end{vmatrix}$ is equal to

a) 2 b) 4 c) 0 d) 1

200. If ω be a complex cube root of unity, then $\begin{vmatrix} 1 & \omega & -\omega^2/2 \\ 1 & 1 & 1 \\ 1 & -1 & 0 \end{vmatrix}$ is equal to

a) 0 b) 1 c) ω d) ω^2

201. $\begin{vmatrix} \log e & \log e^2 & \log e^3 \\ \log e^2 & \log e^3 & \log e^4 \\ \log e^3 & \log e^4 & \log e^5 \end{vmatrix}$ is equal to

a) 0 b) 1 c) $4 \log e$ d) $5 \log e$

202. The value of the determinant, $\begin{vmatrix} \sqrt{13}+\sqrt{3} & 2\sqrt{5} & \sqrt{5} \\ \sqrt{15}+\sqrt{26} & 5 & \sqrt{10} \\ 3+\sqrt{65} & \sqrt{15} & 5 \end{vmatrix}$ is

a) $5(\sqrt{6}-5)$ b) $5\sqrt{3}(\sqrt{6}-5)$ c) $\sqrt{5}(\sqrt{6}-\sqrt{3})$ d) $\sqrt{2}(\sqrt{7}-\sqrt{5})$

203. If $\Delta_1 = \begin{vmatrix} 10 & 4 & 3 \\ 17 & 7 & 4 \\ 4 & -5 & 7 \end{vmatrix}, \Delta_2 = \begin{vmatrix} 4 & x+5 & 3 \\ 7 & x+12 & 4 \\ -5 & x-1 & 7 \end{vmatrix}$ such that $\Delta_1 + \Delta_2 = 0$, is

a) $x = 5$ b) $x = 0$ c) x has no real value d) None of these

204. Let $\Delta = \begin{vmatrix} 1+x_1y_1 & 1+x_1y_2 & 1+x_1y_3 \\ 1+x_2y_1 & 1+x_2y_2 & 1+x_2y_3 \\ 1+x_3y_1 & 1+x_3y_2 & 1+x_3y_3 \end{vmatrix}$, then value of Δ is

a) $x_1x_2x_3 + y_1y_2y_3$ b) $x_1x_2x_3y_1y_2y_3$
c) $x_2x_3y_2y_3 + x_3y_1y_3y_1 + x_1x_2y_1y_2$ d) 0

205. If $\begin{vmatrix} a & a+d & a+2d \\ a^2 & (a+d)^2 & (a+2d)^2 \\ 2a+3d & 2(a+d) & 2a+d \end{vmatrix} = 0$, then

a) $d = 0$ b) $a + d = 0$ c) $d = 0$ or $a + d = 0$ d) None of these

206. Determinant $\begin{vmatrix} b^2-ab & b-c & bc-ac \\ ab-a^2 & a-b & b^2-ab \\ bc-ac & c-a & ab-a^2 \end{vmatrix}$ is equal to

a) $abc(a+b+c)$ b) $3a^2b^2c^2$ c) 0 d) None of these

207. If the system of equations
$bx + ay = c, cx + az = b, cy + bz = a$
has a unique solution, then

a) $abc = 1$ b) $abc = -2$ c) $abc = 0$ d) None of these

208. If ω is a cube root of unity, then $\begin{vmatrix} x+1 & \omega & \omega^2 \\ \omega & x+\omega^2 & 1 \\ \omega^2 & 1 & x+\omega \end{vmatrix}$, is equal to

a) $x^3 + 1$ b) $x^3 + \omega$ c) $x^3 + \omega^2$ d) x^3

209. If A and B are two matrices such that $A + B$ and AB are both defined, then
a) A and B are two matrices not necessarily of same order
b) A and B are square matrices of same order
c) Number of columns of A = Number of rows of B
d) None of these

210. The coefficient of x in $f(x) = \begin{vmatrix} x & 1+\sin x & \cos x \\ 1 & \log(1+x) & 2 \\ x^2 & 1+x^2 & 0 \end{vmatrix}, -1 < x \le 1$, is

a) 1 b) -2 c) -1 d) 0

211. The value of $\begin{vmatrix} a & a^2-bc & 1 \\ b & b^2-ca & 1 \\ c & c^2-ab & 1 \end{vmatrix}$, is

a) 1 b) -1 c) 0 d) $-abc$

212. The value of the determinant $\begin{vmatrix} 1 & \omega^3 & \omega^5 \\ \omega^3 & 1 & \omega^4 \\ \omega^5 & \omega^4 & 1 \end{vmatrix}$, where ω is an imaginary cube root of unity, is

a) $(1-\omega)^2$ b) 3 c) -3 d) None of these

213. Let a, b, c, be positive and not all equal, the value of the

Determinant $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ is

a) Positive b) Negative c) Zero d) None of these

214. If $\begin{vmatrix} -12 & 0 & \lambda \\ 0 & 2 & -1 \\ 2 & 1 & 15 \end{vmatrix} = -360$, then the value of λ, is

a) -1 b) -2 c) -3 d) 4

215. If $\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix}$0 and vectors $(1, a, a^2)$, $(1, b, b^2)$and $(1, c, c^2)$are non-coplanar, then the product abc equals

a) 2 b) -1 c) 1 d) 0

216. ω is an imaginary cube root of unity and

$\begin{vmatrix} x+\omega^2 & \omega & 1 \\ \omega & \omega^2 & 1+x \\ 1 & x+\omega & \omega^2 \end{vmatrix} = 0$, then one of the value of x is

a) 1 b) 0 c) -1 d) 2

217. if x, y, z are in A.P., then the value of the det(A) is, where

$A = \begin{bmatrix} 4 & 5 & 6 & x \\ 5 & 6 & 7 & y \\ 6 & 7 & 8 & z \\ x & y & z & 0 \end{bmatrix}$

a) 0 b) 1 c) 2 d) None of these

218. If $\alpha, \beta, \gamma \in R$, then the determinant $\Delta = \begin{vmatrix} (e^{i\alpha}+e^{-i\alpha})^2 & (e^{i\alpha}-e^{-i\alpha})^2 & 4 \\ (e^{i\beta}+e^{-i\beta})^2 & (e^{i\beta}-e^{-i\beta})^2 & 4 \\ (e^{i\gamma}+e^{-i\gamma})^2 & (e^{i\gamma}-e^{-i\gamma})^2 & 4 \end{vmatrix}$ is

a) Independent of α, β and γ b) Dependent of α, β and γ
c) Independent of α, β only d) Independent of α, β only

219. If $a > 0, b > 0, c > 0$ are respectively the p^{th}, q^{th}, r^{th} terms of a GP, then the value of the determinant

$\begin{vmatrix} \log a & p & 1 \\ \log b & q & 1 \\ \log c & r & 1 \end{vmatrix}$, is

a) 1 b) 0 c) -1 d) None of these

220. The sum of the products of the elements of any row of a determinant A with the cofactors of the corresponding elements is equal to

a) 1 b) 0 c) $|A|$ d) $\frac{1}{2}|A|$

221. If a, b, c, d, e and $f$are in GP, then the value of

$\begin{vmatrix} a^2 & d^2 & x \\ b^2 & e^2 & y \\ c^2 & f^2 & z \end{vmatrix}$

a) Depends on x and y b) Depends on x and z
c) Depends on y and z d) independents on x, y and z

222. The value of $\begin{vmatrix} 1 & \log_x y & \log_x z \\ \log_y x & 1 & \log_y z \\ \log_z x & \log_z y & 1 \end{vmatrix}$ is equal to

a) 0 b) 1 c) xyz d) $\log xyz$

223. The value of the determinant $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1-x & 1 \\ 1 & 1 & 1+y \end{vmatrix}$ is equal to

a) $3-x+y$ b) $(1-x)(1+y)$ c) xy d) $-xy$

224. If $\begin{vmatrix} x & y & z \\ -x & y & z \\ x & -y & z \end{vmatrix} = kxyz$, then k is equal to

a) 1 b) 3 c) 4 d) 2

225. If $x=-5$ is a root of $\begin{vmatrix} 2x+1 & 4 & 8 \\ 2 & 2x & 2 \\ 7 & 6 & 2x \end{vmatrix} = 0$, then the other roots are

a) 3, 3, 5 b) 1, 3, 5 c) 1, 7 d) 2, 7

226. Let a, b, c be positive real numbers. The following system of equations in x, y and z

$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1, \frac{x^2}{a^2}-\frac{y^2}{b^2}+\frac{z^2}{c^2}=1, -\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ has

a) No solution

b) Unique solution

c) Infinitely many solutions

d) Finitely many solutions

227. $\begin{vmatrix} 1+\sin^2\theta & \sin^2\theta & \sin^2\theta \\ \cos^2\theta & 1+\cos^2\theta & \cos^2\theta \\ 4\sin 4\theta & 4\sin 4\theta & 1+4\sin 4\theta \end{vmatrix} = 0$, then $\sin 4\theta$ equals to

a) 1/2 b) 1 c) $-1/2$ d) -1

228. If a, b, c are unequal what is the condition that the value of the determinant, $\Delta \equiv \begin{vmatrix} a & a^2 & a^3+1 \\ b & b^2 & b^3+1 \\ c & c^2 & c^3+1 \end{vmatrix}$ is 0?

a) $1+abc=0$ b) $a+b+c+1=0$

c) $(a-b)(b-c)(c-a)=0$ d) None of these

229. If $\alpha+\beta+\gamma=\pi$, then the value of the determinant

$\begin{vmatrix} e^{2i\alpha} & e^{-i\gamma} & e^{-i\beta} \\ e^{-i\gamma} & e^{2i\beta} & e^{-i\alpha} \\ e^{-i\beta} & e^{-i\alpha} & e^{2i\gamma} \end{vmatrix}$, is

a) 4 b) -4 c) 0 d) None of these

230. If $x^a y^b = e^m, x^c y^d = e^n, \Delta_1 = \begin{vmatrix} m & b \\ n & d \end{vmatrix}, \Delta_2 = \begin{vmatrix} a & m \\ c & n \end{vmatrix}$ and $\Delta_3 = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$, then the values of x and y are respectively

a) $\frac{\Delta_1}{\Delta_3}$ and $\frac{\Delta_2}{\Delta_3}$ b) $\frac{\Delta_2}{\Delta_1}$ and $\frac{\Delta_3}{\Delta_1}$

c) $\log\left(\frac{\Delta_1}{\Delta_3}\right)$ and $\log\left(\frac{\Delta_2}{\Delta_3}\right)$ d) e^{Δ_1/Δ_3} and e^{Δ_2/Δ_3}

231. If $a \neq b \neq c$, then the value of x satisfying the equation

$\begin{vmatrix} 0 & x^2-a & a-b \\ x+a & 0 & x-c \\ x+b & x-c & 0 \end{vmatrix} = 0$ is

a) a b) b c) c d) 0

232. The value of the determinant $\begin{vmatrix} 10! & 11! & 12! \\ 11! & 12! & 13! \\ 12! & 13! & 14! \end{vmatrix}$ is

a) 2(10! 11!) b) 2(10! 13!) c) 2(10! 11! 12!) d) 2(11! 12! 13!)

233. The number of distinct real root of $\begin{vmatrix} \sin x & \cos x & \cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix} = 0$ in the interval $-\frac{\pi}{4} \le x \le \frac{\pi}{4}$ is

a) 0 b) 2 c) 1 d) 3

234. The value of determinant $\begin{vmatrix}(a^x+a^{-x})^2 & (a^x-a^{-x})^2 & 1\\(b^x+b^{-x})^2 & (b^x-b^{-x})^2 & 1\\(c^x+c^{-x})^2 & (c^x-c^{-x})^2 & 1\end{vmatrix}$ is

a) 0 b) $2\,abc$ c) $a^2b^2c^2$ d) None of these

235. The matrix $\begin{bmatrix}\lambda & 1 & 0\\0 & 2 & 3\\0 & 0 & \lambda\end{bmatrix}$ is non- singular

a) For all real values of λ b) Only when $\lambda=\pm\frac{1}{\sqrt{2}}$ c) Only when $\lambda\neq 0$ d) Only when $\lambda=0$

236. If $\begin{vmatrix}a^2 & b^2 & c^2\\(a+1)^2 & (b+1)^2 & (c+1)^2\\(a-1)^2 & (b-1)^2 & (c-1)^2\end{vmatrix}=k\begin{vmatrix}a^2 & b^2 & c^2\\a & b & c\\1 & 1 & 1\end{vmatrix}$,

Then the value of k is

a) 1 b) 2 c) 3 d) 4

237. If $f(x)$, g(x) and $h(x)$ are three polynomials of degree 2 and $\Delta(x)=\begin{vmatrix}f(x) & g(x) & h(x)\\f'(x) & g'(x) & h'(x)\\f''(x) & g''(x) & h''(x)\end{vmatrix}$, then $\Delta(x)$ is polynomial of degree

a) 2 b) 3 c) At most 2 d) At most 3

238. The value of $\begin{vmatrix}x+y & y+z & z+x\\x & y & z\\x-y & y-z & z-x\end{vmatrix}$ is equal to

a) $2(x+y+z)^2$ b) $2(x+y+z)^3$ c) $(x+y+z)^3$ d) 0

239. If $f(x)=\begin{vmatrix}1+a & 1+ax & 1+ax^2\\1+b & 1+bx & 1+bx^2\\1+c & 1+cx & 1+cx^2\end{vmatrix}$, where a,b,c are non-zero constants, then value of $f(10)$is

a) $10\,(b-a)(c-a)$ b) $100\,(b-a)(c-b)(a-c)$

c) $100\,abc$ d) 0

240. The value of λ, if $ax^4+bx^3+cx^2+50x+d=\begin{vmatrix}x^3-14x^2 & -x & 3x+\lambda\\4x+1 & 3x & x-4\\-3 & 4 & 0\end{vmatrix}$, is

a) 0 b) 1 c) 2 d) 3

241. If $\begin{vmatrix}x^2+x & x+1 & x-2\\2x^2+3x-1 & 3x & 3x-3\\x^2+2x+3 & 2x-1 & 2x-1\end{vmatrix}=Ax-12$, then the value of A is

a) 12 b) 23 c) -12 d) 24

242. The value of x obtained from the equation $\begin{vmatrix}x+\alpha & \beta & \gamma\\\gamma & x+\beta & \alpha\\\alpha & \beta & x+\gamma\end{vmatrix}=0$ will be

a) 0 and – $(\alpha+\beta+\gamma)$ b) 0 and $(\alpha+\beta+\gamma)$

c) 1 and $(\alpha-\beta-\gamma)$ d) 0 and $(\alpha^2+\beta^2+\gamma^2)$

243. $\begin{vmatrix}1+ax & 1+bx & 1+cx\\1+a_1x & 1+b_1x & 1+c_1x\\1+a_2x & 1+b_2x & 1+c_2x\end{vmatrix}=A_0+A_1\,x+A_2\,x^2+A_3\,x^3$, then A_1 is equal to

a) abc b) 0 c) 1 d) None of these

244. From the matrix equation $AB=AC$ we can conclude $B=C$ provided that

a) A is singular b) A is non-singular c) A is symmetric d) A is square

245. If $a\neq b$, then the system of equation

$ax+by+bz=0$

$bx+ay+bz=0$

$bx+by+az=0$

Will have a non-trivial solution, is

a) $a+b=0$ b) $a+2b=0$ c) $2a+b=0$ d) $a+4b=0$

246. If ω is an imaginary cube root of unity, then the value of

$\begin{vmatrix} a & b\omega^2 & a\omega \\ b\omega & c & b\omega^2 \\ c\omega^2 & a\omega & c \end{vmatrix}$, is

a) $a^3+b^3+c^3$ b) a^2b-b^2c c) 0 d) $a^3+b^3+c^3-3abc$

247. The value of determinant $\begin{vmatrix} a+b & a+2b & a+3b \\ a+2b & a+3b & a+4b \\ a+4b & a+5b & a+6b \end{vmatrix}$ is

a) $a^2+b^2+c^2-3abc$ b) $3ab$ c) $3a+5b$ d) 0

248. The value of the determinant $\begin{vmatrix} y+z & x & x \\ y & z+x & y \\ z & z & x+y \end{vmatrix}$ is equal to

a) $6xyz$ b) xyz c) $4xyz$ d) $xy+yz+zx$

249. If $\Delta_1 = \begin{vmatrix} x & b & b \\ a & x & b \\ a & a & x \end{vmatrix}$ and $\Delta_2 = \begin{vmatrix} x & b \\ a & x \end{vmatrix}$, then

a) $\Delta_1 = 3(\Delta_2)^2$ b) $\frac{d}{dx}(\Delta_1) = 3\,\Delta_2$ c) $\frac{d}{dx}(\Delta_1) = 3\,\Delta_2^2$ d) $\Delta_1 = 3\,(\Delta_2)^{3/2}$

250. For positive numbers x, y, z (other than unity) the numerical value of the determinant

$\begin{vmatrix} 1 & \log_x y & \log_x z \\ \log_y x & 3 & \log_y z \\ \log_z x & \log_z y & 5 \end{vmatrix}$, is

a) 0 b) $\log x \log y \log z$ c) 1 d) 8

251. The value of $\begin{vmatrix} 1990 & 1991 & 1992 \\ 1991 & 1992 & 1993 \\ 1992 & 1993 & 1994 \end{vmatrix}$ is

a) 1992 b) 1993 c) 1994 d) 0

252. If $\begin{vmatrix} a+x & a-x & a-x \\ a-x & a+x & a-x \\ a-x & a-x & a+x \end{vmatrix} = 0$, then x is equal to

a) $0, 2a$ b) $a, 2a$ c) $0, 3a$ d) None of these

253. The determinant $\Delta = \begin{vmatrix} b & c & b\,\alpha+c \\ c & d & c\,\alpha+d \\ b\,\alpha+c & c\,\alpha+d & a\,\alpha^3-c\,\alpha \end{vmatrix}$ is equal to zero, if

a) b, c, d are in A.P.

b) b, c, d are in G.P.

c) b, c, d are in H.P.

d) α is a root of $ax^3+bx^2-cx-d=0$

254. If $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 5$, then the value of $\begin{vmatrix} b_2c_3-b_3c_2 & c_2a_3-c_3a_2 & a_2b_3-c_3b_2 \\ b_3c_1-b_1c_3 & c_3a_1-c_1a_3 & a_3b_1-a_1b_3 \\ b_1c_2-b_2c_1 & c_1a_2-c_2a_1 & a_1b_2-a_2b_1 \end{vmatrix}$ is

a) 5 b) 25 c) 125 d) 0

255. The determinant $\Delta = \begin{vmatrix} a^2+x^2 & ab & ac \\ ab & b^2+x^2 & bc \\ ac & bc & c^2+x^2 \end{vmatrix}$ is divisible by

a) x^5 b) x^4 c) x^4+1 d) x^4-1

256. If $\Delta_a = \begin{vmatrix} a-1 & n & 6 \\ (a-1)^2 & 2\,n^2 & 4\,n-2 \\ (a-1)^3 & 3\,n^3 & 3\,n^2-3\,n \end{vmatrix}$, then $\sum_{a=1}^{n} \Delta_a$ is equal to

a) 0 b) 1 c) $\left\{\frac{n(n+1)}{2}\right\}\left\{\frac{a(a+1)}{2}\right\}$ d) None of these

257. Let the determinant of a 3×3 matrix A be 6, then B is a matrix defined by $B = 5A^2$. Then, determinant of B is

a) 180 b) 100 c) 80 d) None of These

258. The coefficient of x in $f(x)=\begin{vmatrix} x & 1+\sin x & \cos x \\ 1 & \log(1+x) & 2 \\ x^2 & 1+x^2 & 0 \end{vmatrix}, -1 < x \le 1$, is

a) 1 b) -2 c) -1 d) 0

259. The value of $\begin{vmatrix} 1 & 1 & 1 \\ bc & ca & ab \\ b+c & c+a & a+b \end{vmatrix}$ is

a) 1 b) 0 c) $(a-b)(b-c)(c-a)$ d) $(a+b)(b+c)(c+a)$

260. A factor of $\Delta(x)=\begin{vmatrix} x^3+1 & 2x^4+3x^2 & 3x^5+4x \\ 2 & 5 & 7 \\ 3 & 14 & 19 \end{vmatrix}$ is

a) x b) $(x-1)^2$ c) $(x+1)^2$ d) None of these

261. If $p\lambda^4+q\lambda^3+q\lambda^2+s\lambda+t=\begin{vmatrix} b^2+c^2 & a^2+\lambda & a^2+\lambda \\ b^2+\lambda & c^2+a^2 & b^2+\lambda \\ c^2+\lambda & c^2+\lambda & a^2+b^2 \end{vmatrix}$ is an identity in λ, where p, q, r, s, t are constants, then the value of t is

a) 1 b) 2 c) 0 d) None of these

262. The value of the determinant $\begin{vmatrix} 10! & 11! & 12! \\ 11! & 12! & 13! \\ 12! & 13! & 14! \end{vmatrix}$ is

a) 2 (10! 11!) b) 2 (10! 13!) c) 2 (10! 11! 12!) d) 2 (11! 12! 13!)

263. If $A_i=\begin{bmatrix} a^i & b^i \\ b^i & a^i \end{bmatrix}$ and if $|a|<1, |b|<1$,then $\sum_{i=1}^{\infty}\det(A_i)$ is equal to

a) $\frac{a^2}{(1-a)^2}-\frac{b^2}{(1-b)^2}$ b) $\frac{a^2-b^2}{(1-a)^2(1-b^2)}$ c) $\frac{a^2}{(1-a)^2}+\frac{b^2}{(1-b)^2}$ d) $\frac{a^2}{(1+a)^2}-\frac{b^2}{(1+b)^2}$

264. If $\begin{bmatrix} 1 & 2 & x \\ 4 & -1 & 7 \\ 2 & 4 & -6 \end{bmatrix}$ is a singular matrix, then x is equal to

a) 0 b) 1 c) -3 d) 3

265. The value of $\begin{vmatrix} x & p & q \\ p & x & q \\ p & q & x \end{vmatrix}$ is

a) $x(x-p)(x-q)$ b) $(x-p)(x-q)(x+p+q)$

c) $(p-q)(x-q)(x-p)$ d) $pq(x-p)(x-q)$

266. The roots of the equation $\begin{vmatrix} 1 & 4 & 20 \\ 1 & -2 & 5 \\ 1 & 2x & 5x^2 \end{vmatrix}=0$ are

a) $-1,-2$ b) $-1,2$ c) $1,-2$ d) $1,2$

267. If $\begin{vmatrix} 6i & -3i & 1 \\ 4 & 3i & -1 \\ 40 & 3 & i \end{vmatrix}=x+iy$, then

a) $x=3, y=1$ b) $x=1, y=3$ c) $x=0, y=3$ d) $x=0, y=0$

268. The determinant

$\Delta=\begin{vmatrix} \cos(\alpha+\beta) & -\sin(\alpha+\beta) & \cos 2\beta \\ \sin\alpha & \cos\alpha & \sin\beta \\ -\sin\alpha & \sin\alpha & \cos\beta \end{vmatrix}$ is independent of

a) α b) β c) α and β d) Neither α nor β

269. $\begin{vmatrix} x+1 & x+2 & x+a \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix}=0$,then a, b, c are

a) In GP b) In HP c) Equal d) In AP

270. If $1+\frac{1}{a}+\frac{1}{c}+\frac{1}{c}=0$, then $\Delta=\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix}$ is equal to

a) 0 b) abc c) $-abc$ d) None of these

271. If $a \neq b \neq c$, the value of x which satisfies the equation

$$\begin{vmatrix} 0 & x-a & x-b \\ x+a & 0 & x-c \\ x+b & x+c & 0 \end{vmatrix} = 0, \text{ is}$$

a) $x = 0$ b) $x = a$ c) $x = b$ d) $x = c$

272. If $D_r = \begin{vmatrix} r & 1 & \frac{n(n+1)}{2} \\ 2r-1 & 4 & n^2 \\ 2^{r-1} & 5 & 2^n - 1 \end{vmatrix}$, then the value of $\sum_{r=0}^{n} D_r$ is

a) 0 b) 1 c) $\frac{n(n+1)(2n+1)}{6}$ d) None of these

273. If $f(\alpha) = \begin{vmatrix} 1 & \alpha & \alpha^2 \\ \alpha & \alpha^2 & 1 \\ \alpha^2 & 1 & \alpha \end{vmatrix}$, then $f(\sqrt[3]{3})$ is equal to

a) 1 b) -4 c) 4 d) 2

274. The value of the determinant $\Delta = \begin{vmatrix} 2a_1b_1 & a_1b_2 + a_2b_1 & a_1b_3 + a_3b_1 \\ a_1b_2 + a_2b_1 & 2a_2b_2 & a_2b_3 + a_3b_2 \\ a_1b_3 + a_3b_1 & a_3b_2 + a_2b_3 & 2a_3b_3 \end{vmatrix}$ is

a) 1 b) $2a_1a_2a_3b_1b_2b_3$ c) 0 d) $a_1a_2a_3b_1b_2b_3$

275. If $A = \begin{bmatrix} 3 & 2 & 4 \\ 1 & 2 & 1 \\ 3 & 2 & 6 \end{bmatrix}$ and A_{ij} are the cofactors of a_{ij}, then

$a_{11}A_{11} + a_{12}A + a_{13}A_{13}$ is equal to

a) 8 b) 6 c) 4 d) 0

276. The equation $\begin{vmatrix} x-a & x-b & x-c \\ x-b & x-c & x-a \\ x-c & x-a & x-b \end{vmatrix} = 0$, where a, b, c are different, is satisfied by

a) $x = 0$ b) $x = a$ c) $x = \frac{1}{3}(a+b+c)$ d) $a = a+b+c$

277. $\begin{vmatrix} x & p & q \\ p & x & q \\ q & q & x \end{vmatrix} =$

a) $(x+p)(x+q)(x-p-q)$

b) $(x-p)(x-q)(x+p+q)$

c) $(x-p)(x-q)(x-p-q)$

d) $(x+p)(x+q)(x+p+q)$

278. If $f(x) = \begin{vmatrix} 1 & 1 & 1 \\ 2x & (x-1) & x \\ 3x(x-1) & (x-1)(x-2) & x(x-1) \end{vmatrix}$, then $f(50)$ is equal to

a) 0 b) 1 c) 100 d) -100

DETERMINANTS

: ANSWER KEY :

1)	b	2)	a	3)	b	4)	b	141)	c	142)	d	143)	c	144)	b
5)	c	6)	b	7)	c	8)	c	145)	b	146)	c	147)	d	148)	b
9)	b	10)	a	11)	b	12)	d	149)	a	150)	d	151)	a	152)	b
13)	d	14)	d	15)	b	16)	a	153)	d	154)	c	155)	a	156)	a
17)	c	18)	a	19)	b	20)	c	157)	b	158)	d	159)	b	160)	c
21)	c	22)	c	23)	b	24)	a	161)	a	162)	d	163)	c	164)	d
25)	b	26)	b	27)	c	28)	d	165)	d	166)	c	167)	a	168)	c
29)	d	30)	a	31)	c	32)	b	169)	c	170)	d	171)	b	172)	a
33)	d	34)	a	35)	c	36)	a	173)	c	174)	c	175)	b	176)	d
37)	a	38)	a	39)	b	40)	c	177)	b	178)	b	179)	a	180)	d
41)	d	42)	b	43)	b	44)	d	181)	d	182)	a	183)	c	184)	a
45)	d	46)	a	47)	b	48)	c	185)	c	186)	b	187)	a	188)	a
49)	d	50)	a	51)	c	52)	b	189)	c	190)	d	191)	c	192)	d
53)	c	54)	b	55)	c	56)	a	193)	a	194)	a	195)	c	196)	a
57)	d	58)	d	59)	d	60)	c	197)	d	198)	d	199)	c	200)	a
61)	b	62)	c	63)	d	64)	a	201)	a	202)	b	203)	a	204)	d
65)	c	66)	a	67)	b	68)	b	205)	c	206)	c	207)	c	208)	d
69)	b	70)	c	71)	c	72)	b	209)	b	210)	b	211)	c	212)	b
73)	b	74)	a	75)	b	76)	a	213)	b	214)	c	215)	b	216)	b
77)	d	78)	a	79)	a	80)	b	217)	a	218)	a	219)	b	220)	c
81)	c	82)	c	83)	b	84)	b	221)	d	222)	a	223)	d	224)	c
85)	a	86)	a	87)	b	88)	a	225)	b	226)	b	227)	c	228)	a
89)	d	90)	a	91)	c	92)	b	229)	b	230)	d	231)	d	232)	c
93)	c	94)	d	95)	a	96)	c	233)	c	234)	a	235)	c	236)	d
97)	b	98)	a	99)	b	100)	b	237)	c	238)	d	239)	d	240)	c
101)	a	102)	b	103)	a	104)	a	241)	d	242)	a	243)	b	244)	b
105)	b	106)	b	107)	b	108)	d	245)	b	246)	c	247)	d	248)	c
109)	a	110)	d	111)	d	112)	b	249)	b	250)	d	251)	d	252)	c
113)	a	114)	a	115)	b	116)	a	253)	b	254)	b	255)	b	256)	a
117)	d	118)	d	119)	d	120)	a	257)	d	258)	b	259)	c	260)	b
121)	c	122)	d	123)	c	124)	a	261)	d	262)	c	263)	b	264)	c
125)	a	126)	d	127)	a	128)	d	265)	b	266)	b	267)	d	268)	a
129)	d	130)	a	131)	b	132)	a	269)	d	270)	a	271)	a	272)	a
133)	b	134)	b	135)	c	136)	c	273)	b	274)	c	275)	a	276)	c
137)	a	138)	d	139)	d	140)	c	277)	b	278)	a				

DETERMINANTS

: HINTS AND SOLUTIONS :

2 **(a)**

Given $\begin{vmatrix} x & 2 & -1 \\ 2 & 5 & x \\ -1 & 2 & x \end{vmatrix} = 0$

$\Rightarrow \begin{vmatrix} x & 2 & -1 \\ 2 & 5 & x \\ -3 & -3 & 0 \end{vmatrix} = 0 \quad [R_3 \to R_3 - R_2]$

$\Rightarrow -1(-6+15) - x[-3x+6] = 0$

$\Rightarrow \quad x^2 - 2x - 3 = 0$

$\Rightarrow x = 3, -1$

3 **(b)**

$\begin{vmatrix} 441 & 442 & 443 \\ 445 & 446 & 447 \\ 449 & 450 & 451 \end{vmatrix} = \begin{vmatrix} 441 & 1 & 1 \\ 445 & 1 & 1 \\ 449 & 1 & 1 \end{vmatrix}$

$C_2 \to C_2 - C_1$

$C_3 \to C_3 - C_2$

=0 [$\because$ two columns are identical]

4 **(b)**

Given, $f(\alpha) = \begin{vmatrix} 1 & \alpha & \alpha^2 \\ \alpha & \alpha^2 & 1 \\ \alpha^2 & 1 & \alpha \end{vmatrix}$

$= 1(\alpha^3 - 1) - \alpha(\alpha^2 - \alpha^2) + \alpha^2(\alpha - \alpha^4)$

$= \alpha^3 - 1 - 0 + \alpha^3 - \alpha^6$

$\Rightarrow f(\sqrt[3]{3}) = 3 - 1 - 0 + 3 - 3^2$

$= 6 - 10 = -4$

5 **(c)**

Let the first term and common difference of an AP are A and D respectively.

$\therefore a = A + (p-1)D, b = A + (q-1)D,$

and $c = A + (r-1)D$

Now, $\begin{vmatrix} a & p & 1 \\ b & q & 1 \\ c & r & 1 \end{vmatrix} = \begin{vmatrix} A+(p-1)D & p & 1 \\ A+(q-1)D & q & 1 \\ A+(r-1)D & r & 1 \end{vmatrix}$

Applying $C_1 \to C_1 - DC_2 + DC_3$

$= \begin{vmatrix} A & p & 1 \\ A & q & 1 \\ A & r & 1 \end{vmatrix} = A \begin{vmatrix} 1 & p & 1 \\ 1 & q & 1 \\ 1 & r & 1 \end{vmatrix} = 0$ ($\because$ two columns are identical)

6 **(b)**

Minor of $(-4) = \begin{vmatrix} -2 & 3 \\ 8 & 9 \end{vmatrix} = -42$

Minor of 9$=\begin{vmatrix} -1 & -2 \\ -4 & -5 \end{vmatrix} = -3$

Cofactor of$(-4) = (-1)^{2+1} . \begin{vmatrix} -2 & 3 \\ 8 & 9 \end{vmatrix} = 42$

and cofactor of 9 $= (-1)^{3+3} . \begin{vmatrix} -1 & -2 \\ -4 & -5 \end{vmatrix} = -3$

7 **(c)**

Given, α, β and γ are the cube roots of unity, then assume

$\alpha = 1, \beta = \omega$ and $\gamma = \omega^2$.

$\therefore \begin{vmatrix} e^\alpha & e^{2\alpha} & (e^{3\alpha} - 1) \\ e^\beta & e^{2\beta} & (e^{3\beta} - 1) \\ e^\gamma & e^{2\gamma} & (e^{3\gamma} - 1) \end{vmatrix}$

$= \begin{vmatrix} e^\alpha & e^{2\alpha} & e^{3\alpha} \\ e^\beta & e^{2\beta} & e^{3\beta} \\ e^\gamma & e^{2\gamma} & e^{3\gamma} \end{vmatrix} + \begin{vmatrix} e^\alpha & e^{2\alpha} & -1 \\ e^\beta & e^{2\beta} & -1 \\ e^\gamma & e^{2\gamma} & -1 \end{vmatrix}$

$= e^\alpha e^\beta e^\gamma \begin{vmatrix} 1 & e^\alpha & e^{2\alpha} \\ 1 & e^\beta & e^{2\beta} \\ 1 & e^\gamma & e^{2\gamma} \end{vmatrix} - \begin{vmatrix} 1 & e^\alpha & e^{2\alpha} \\ 1 & e^\beta & e^{2\beta} \\ 1 & e^\gamma & e^{2\gamma} \end{vmatrix}$

$= \begin{vmatrix} 1 & e^\alpha & e^{2\alpha} \\ 1 & e^\beta & e^{2\beta} \\ 1 & e^\gamma & e^{2\gamma} \end{vmatrix} [e^\alpha e^\beta e^\gamma - 1] = 0$

$(\because e^\alpha e^\beta e^\gamma = e^{1+\omega+\omega^2} = e^0 = 1)$

8 **(c)**

Applying $C_1 \to C_1 + C_2 + C_3$, we obtain

$-x \begin{vmatrix} 1 & -6 & 3 \\ 1 & 3-x & 3 \\ 1 & 3 & -6-x \end{vmatrix} = 0$

$\Rightarrow -x \begin{vmatrix} 1 & -6 & 3 \\ 0 & 9-x & 0 \\ 0 & 9 & -9-x \end{vmatrix} = 0$

$\begin{bmatrix} \text{Applying } R_2 \to R_2 - R_1, \\ R_3 \to R_3 - R_1 \end{bmatrix}$

$\Rightarrow -x(9-x)(-9-x) = 0 \Rightarrow x = 0, 9, -9$

9 **(b)**

$\begin{vmatrix} \log_3 512 & \log_4 3 \\ \log_3 8 & \log_4 9 \end{vmatrix} \times \begin{vmatrix} \log_2 3 & \log_8 3 \\ \log_3 4 & \log_3 4 \end{vmatrix}$

$= (\log_3 512 \times \log_4 9 - \log_4 3 \log_3 8) \times (\log_2 3 \times \log_3 4 - \log_8 3 \times \log_3 4)$

$= \left(\frac{\log 512}{\log 3} \times \frac{\log 9}{\log 4} - \frac{\log 3}{\log 4} \times \frac{\log 8}{\log 3} \right) \times \left(\frac{\log 3}{\log 2} \times \frac{\log 4}{\log 3} \times \frac{\log 3}{\log 8} \times \frac{\log 4}{\log 3} \right)$

$= \left(\frac{\log 2^9}{\log 3} \times \frac{\log 3^2}{\log 2^2} \times \frac{\log 2^3}{\log 2^2} \right) \times \left(\frac{\log 2^2}{\log 2} - \frac{\log 2^2}{\log 2^3} \right)$

$= \left(\frac{9 \times 2}{2} - \frac{3}{2} \right) \times \left(2 - \frac{2}{3} \right) = \frac{15}{2} \times \frac{4}{3} = 10$

10 **(a)**

Le t$\Delta = \begin{vmatrix} a^2 & a & 1 \\ \cos(nx) & \cos(n+1)x & \cos(n+2)x \\ \sin(nx) & \sin(n+1)x & \sin(n+2)x \end{vmatrix}$

Since, $\cos(nx) + \cos(n+2)x = 2\cos(n+1)x \cos x$

and $\sin(nx) + \sin(n+2)x = 2\sin(n+1)x \cos x$

Applying $C_1 \to C_1 - 2\cos x \cdot C_2 + C_3$

$\therefore \Delta$

$$= \begin{vmatrix} a^2 - 2a\cos x + 1 & a & 1 \\ 0 & \cos(n+1)x & \cos(n+2)x \\ 0 & \sin(n+1)x & \sin(n+2)x \end{vmatrix}$$

$= (a^2 - 2a\cos x + 1)[\cos(n+1)x \sin(n+2)x - \cos(n+2)x \sin(n+1)x]$

$= (a^2 - 2a\cos x + 1)\sin x$

$\therefore \Delta$ is independent of n.

11 **(b)**

Given $\begin{vmatrix} x+1 & 2x+1 & 3x+1 \\ 2x & 4x+3 & 6x+3 \\ 4x+4 & 6x+4 & 8x+4 \end{vmatrix} = 0$

$\Rightarrow 2\begin{vmatrix} 0 & x & 2x \\ 2x & 4x+3 & 6x+3 \\ 2x+2 & 3x+2 & 4x+2 \end{vmatrix} = 0$

[Using $(R_1 \to 2R_1 - R_3)$]

$\Rightarrow 2\begin{vmatrix} 0 & x & 0 \\ 2x & 4x+3 & -2x-3 \\ 2x+2 & 3x+2 & -2x-2 \end{vmatrix} = 0$

[Using $(C_3 \to C_3 - 2C_2)$]

$\Rightarrow -4x[2x^2 + 2x - (2x+3)(x+1)] = 0$

$\Rightarrow -4x[2x^2 + 2x - (2x^2 + 5x + 3)] = 0$

$\Rightarrow 4x(3x+3) = 0$

$\Rightarrow x + 1 = 0$ $[\because x \neq 0 \text{ given}]$

13 **(d)**

$\begin{vmatrix} a & b-y & c-z \\ a-x & b & c-z \\ a-x & b-y & c \end{vmatrix} = 0$

$\Rightarrow \begin{vmatrix} a & b-y & c-z \\ -x & y & 0 \\ 0 & -y & z \end{vmatrix} = 0$

(Using $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_2$)

$\Rightarrow a(yz) + x(bz - yz + cy - yz) = 0$

$\Rightarrow ayz + bzx + cyx = 2xyz$

$\Rightarrow \frac{a}{x} + \frac{b}{y} + \frac{c}{z} = 2$

14 **(d)**

We have,

$\begin{vmatrix} p & b & c \\ a & q & c \\ a & b & r \end{vmatrix} = 0$

$\Rightarrow \begin{vmatrix} p & b & c \\ a-p & q-b & 0 \\ 0 & b-q & r-c \end{vmatrix} = 0$

$\begin{bmatrix} \text{Applying } R_3 \to R_3 - R_2 \\ \text{and } R_2 \to R_2 - R_1 \end{bmatrix}$

$\Rightarrow \begin{vmatrix} \frac{p}{p-a} & \frac{b}{q-b} & \frac{c}{r-c} \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{vmatrix} = 0$

$\Rightarrow \frac{p}{p-a} + \frac{b}{q-b} + \frac{c}{r-c} = 0$

$\Rightarrow \frac{p}{p-a} + \left(\frac{q}{q-b} - 1\right) + \left(\frac{r}{r-c} - 1\right) = 0$

$\Rightarrow \frac{p}{p-a} + \frac{q}{q-b} + \frac{r}{r-c} = 2$

15 **(b)**

We have,

$\Delta = \begin{vmatrix} a & a+b & a+2b \\ a+2b & a & a+b \\ a+b & a+2b & a \end{vmatrix}$

$\Rightarrow \Delta = \begin{vmatrix} 3a+3b & a+b & a+2b \\ 3a+3b & a & a+b \\ 3a+3b & a+2b & a \end{vmatrix}$ Applying $C_1 \to C_1 + C_2 + C_3$

$\Rightarrow \Delta = 3(a+b)\begin{vmatrix} 1 & a+b & a+2b \\ 1 & a & a+b \\ 1 & a+2b & a \end{vmatrix}$

$\Rightarrow \Delta = 3(a+b)\begin{vmatrix} 1 & a+b & a+2b \\ 0 & -b & -b \\ 0 & b & -2b \end{vmatrix}$

Applying $R_2 \to R_2 - R_1$

$R_3 \to R_3 - R_1$

$\Rightarrow \Delta = 3(a+b)(3b^2) = 9b^2(a+b)$

16 **(a)**

Applying $C_1 \to C_1 + C_2$,we get

$\Rightarrow \begin{vmatrix} 2 & \cos^2\theta & 4\sin 4\theta \\ 2 & 1+\cos^2\theta & 4\sin 4\theta \\ 1 & \cos^2\theta & 1 + 4\sin 4\theta \end{vmatrix} = 0$

$\Rightarrow \begin{vmatrix} 2 & \cos^2\theta & 4\sin 4\theta \\ 0 & 1 & 0 \\ 1 & \cos^2\theta & 1 + 4\sin 4\theta \end{vmatrix} = 0$

$[R_2 \to R_2 - R_1]$

$\Rightarrow (2 + 4\sin 4\theta) = 0$

$\Rightarrow \sin 4\theta = -\frac{1}{2} = -\sin\frac{\pi}{6}$

$\Rightarrow 4\theta = n\pi + (-1)^n\left(-\frac{\pi}{6}\right)$

$\therefore$ The value of θ between 0 and $\frac{\pi}{2}$ will be $\frac{7\pi}{24}$ and $\frac{11\pi}{24}$

17 **(c)**

We have, $a_i^2 + b_i^2 + c_i^2 = 1$

and $a_i a_j + b_i b_j + c_i c_j = 0$ for $(i = 1,2,3)$

$\therefore \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}^2 = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$

$= \begin{vmatrix} a_1^2 + b_1^2 + c_1^2 & a_1a_2 + b_1b_2 + c_1c_2 & a_1a_3 + b_1b_3 + c \\ a_2a_1 + b_2b_1 + c_2c_1 & a_2^2 + b_2^2 + c_2^2 & a_1a_3 + b_1b_3 + c \\ a_3a_1 + b_3b_1 + c_3c_1 & a_3a_2 + b_3b_2 + c_3c_2 & a_3^2 + b_3^2 + c_3^2 \end{vmatrix}$

$= \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = 1$

18 **(a)**

We have,

$\alpha = 2, \beta = 2\omega$ and $\gamma = 2\omega^2 \Rightarrow \alpha + \beta + \gamma = 0$

Now,

$$\begin{vmatrix}\alpha & \beta & \gamma\\ \beta & \gamma & \alpha\\ \gamma & \alpha & \beta\end{vmatrix}$$

$= \begin{vmatrix}\alpha+\beta+\gamma & \beta & \gamma\\ \alpha+\beta+\gamma & \gamma & \alpha\\ \alpha+\beta+\gamma & \alpha & \beta\end{vmatrix}$ Applying $C_1 \to C_1 + C_2 + C_3$

$= \begin{vmatrix}0 & \beta & \gamma\\ 0 & \gamma & \alpha\\ 0 & \alpha & \beta\end{vmatrix} = 0 \qquad [\because \alpha+\beta+\gamma = 0]$

19 **(b)**

Take a, b, c common from R_1, R_2, R_3 respectively,

$$\therefore \Delta = abc \begin{vmatrix}\frac{1}{a}+1 & \frac{1}{a} & \frac{1}{a}\\ \frac{1}{b}+1 & \frac{1}{b}+2 & \frac{1}{b}\\ \frac{1}{c}+1 & \frac{1}{c}+1 & \frac{1}{c}+3\end{vmatrix}$$

Applying $R_1 \to R_1 + R_2 + R_3$

$$\Delta = abc\left(3+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)\begin{vmatrix}1 & 1 & 1\\ 1+\frac{1}{b} & 2+\frac{1}{b} & \frac{1}{b}\\ 1+\frac{1}{c} & 1+\frac{1}{c} & 3+\frac{1}{c}\end{vmatrix}$$

Now, applying $C_3 \to C_3 - C_2$ and $C_2 \to C_2 - C_1$ and on expanding, we get

$\Delta = 2abc\left[3+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right] = 0$

$\because a \neq 0, b \neq 0, c \neq 0$

$\therefore a^{-1} + b^{-1} + c^{-1} = -3$

20 **(c)**

On expanding the given determinant, we obtain

$2x^3 + 2x(ac - ab - bc) = 0 \Rightarrow x = 0$

23 **(b)**

Given, A is a square matrix and $AA^T = I = A^T A$

$\Rightarrow |AA^T| = |I| = |A^T A|$

$\Rightarrow |A||A^T| = I = |A^T||A|$

$\Rightarrow |A|^2 = 1 \qquad [\because |A^T| = |A|]$

$\Rightarrow |A| = \pm 1$

25 **(b)**

We have,

$\begin{vmatrix}y+z & x & y\\ z+x & z & x\\ x+y & y & z\end{vmatrix} = k(x+y+z)(x-z)^2$

LHS$= (x+y+z)\begin{vmatrix}2 & 1 & 1\\ z+x & z & x\\ x+y & y & z\end{vmatrix}$ $(R_1 \to R_1 + R_2 + R_3)$

$= (x+y+z)\begin{vmatrix}1 & 1 & 1\\ x & z & x\\ x & y & z\end{vmatrix}$

$= (x+y+z)\{1(z^2 - xy) - 1(xz - x^2) + 1(xy - xz)\}$

$= (x+y+z)(x^2 + z^2 - 2xz)$

$\Rightarrow (x+y+z)(x-z)^2 = k(x+y+z)(x-z)^2$ (given)

$\Rightarrow k = 1$

26 **(b)**

$\det(2A) = 2^4 \det(A) = 16\det(A)$

27 **(c)**

$\because \det(M_r) = r^2 - (r-1)^2 = 2r - 1$

$\therefore \det(M_1) + \det(M_2) + \ldots + \det(M^{2008})$

$= 1 + 3 + 5 + \ldots + 4015$

$= \frac{2008}{2}[2 + (2008-1)2]$

$= 2008(2008) = (2008)^2$

28 **(d)**

Let A and R be the first term and common ratio respectively.

$\therefore l = AR^{p-1}$

$\Rightarrow \log l = \log A + (p-1)\log R$

$m = AR^{q-1}$

$\Rightarrow \log m = \log A + (q-1)\log R$

and $n = AR^{r-1}$

$\Rightarrow \log n = \log A + (r-1)\log R$

Now, $\begin{vmatrix}\log l & p & 1\\ \log m & q & 1\\ \log n & r & 1\end{vmatrix} =$

$\begin{vmatrix}\log A + (p-1)\log R & p & 1\\ \log A + (q-1)\log R & q & 1\\ \log A + (r-1)\log R & r & 1\end{vmatrix}$

On multiplying R_1, R_2 and R_3 by $(q-r), (r-p)$ and $(p-q)$ and adding $R_1 + R_2 + R_3$, we get

$= (q - r + r - p + p - q).\log A + \{(q-r)(p-1) + (r-p)(q-1) + (p-q)(r-1)\}\log R$

$= 0$

29 **(d)**

Since, the given matrix is singular.

$\therefore \begin{bmatrix}5 & 10 & 3\\ -2 & -4 & 6\\ -1 & -2 & b\end{bmatrix} = 0$

$\Rightarrow 5(-4b + 12) - 10(-2b + 6) + 3(4 - 4) = 0$

$\Rightarrow -20b + 60 + 20b - 60 = 0$

$\Rightarrow 0(b) = 0$

$\therefore$ The given matrix is singular for any value of b

31 **(c)**

Given, $\begin{vmatrix}x^n & x^{n+2} & x^{n+3}\\ y^n & y^{n+2} & y^{n+3}\\ z^n & z^{n+2} & z^{n+3}\end{vmatrix}$

$= (y-z)(z-x)(x-y)\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)$

The degree of determinant

$= n + (n+2) + (n+3) = 3n + 5$

and the degree of RHS=2

$\therefore 3n+5=2 \Rightarrow n=-1$

32 **(b)**

Since , $\begin{vmatrix} a & b & a\alpha+b \\ b & c & b\alpha+c \\ a\alpha+b & b\alpha+c & 0 \end{vmatrix}=0$

Applying $R_3 \to R_3-(\alpha R_1+R_2)$

$\Rightarrow \begin{vmatrix} a & b & a\alpha+b \\ b & c & b\alpha+c \\ 0 & 0 & -a\alpha^2-2b\alpha-c \end{vmatrix}=0$

$\Rightarrow -(a\alpha^2+2b\alpha+c)(ac+b^2)=0$

$\Rightarrow b^2=ac$

Hence, a, b, and c are in GP.

33 **(d)**

The system of equations

$k\,x+y+z=1$

$x+k\,y+z=k$

$x+y+k\,z=k^2$

Is inconsistent, if

$\Delta=\begin{vmatrix} k & 1 & 1 \\ 1 & k & 1 \\ 1 & 1 & k \end{vmatrix}=0$ and one of the $\Delta_1, \Delta_2\Delta_3$ is non-zero, where

$\Delta_1=\begin{vmatrix} 1 & 1 & 1 \\ k & k & 1 \\ k^2 & 1 & k \end{vmatrix}, \Delta_2=\begin{vmatrix} k & 1 & 1 \\ 1 & k & 1 \\ 1 & k^2 & k \end{vmatrix}, \Delta_3$

$=\begin{vmatrix} k & 1 & 1 \\ 1 & k & k \\ 1 & 1 & k^2 \end{vmatrix}$

We have, $\Delta=(k+2)(k-1)^2, \Delta_1=-(k+1)(k-1)^2$

$\Delta_2=-k(k-1)^2, \Delta_3=(k+1)^2(k-1)^2$

Clearly, for $k=-2$, we have

$\Delta=0$ and $\Delta_1, \Delta_2, \Delta_3$ are non-zero. Therefore, $k=-2$

34 **(a)**

We have,

$\Delta=\begin{vmatrix} a & a+b & a+b+c \\ 3a & 4a+3b & 5a+4b+3c \\ 6a & 9a+6b & 11a+9b+6c \end{vmatrix}$

Applying $R_2 \to R_2-3R_1, R_3 \to R_3-2R_2$

$=\begin{vmatrix} a & a & a+b+c \\ 0 & a & 2a+b \\ 0 & a & a+b \end{vmatrix}$

$=a[a^2+ab-2a^2-ab]$

$=-a^3=i \quad (\because a=i, \text{given})$

35 **(c)**

LHS$=\begin{vmatrix} a+b & b+c & c+a \\ b+c & c+a & a+b \\ c+a & a+b & b+c \end{vmatrix}$

The determinant can be written sum of $2\times2\times2=8$ determinants of which 6 are reduces to zero because of their two rows are identical.

$\therefore$ LHS$=2\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$

36 **(a)**

$\begin{vmatrix} \alpha & -\beta & 0 \\ 0 & \alpha & \beta \\ \beta & q & x \end{vmatrix}=0 \Rightarrow \alpha^3-\beta^3=0$

$\Rightarrow \left(\frac{\alpha}{\beta}\right)^3=1 \Rightarrow \frac{\alpha}{\beta}$ is one of the cube roots of unity.

37 **(a)**

Applying $R_3 \to R_3-\alpha\,R_1-R_2$, we get

$\Delta=\begin{vmatrix} b & c & a\,\alpha+b \\ c & d & c\,\alpha+d \\ 0 & 0 & a\,\alpha^3+b\,\alpha^2+c\,\alpha+d \end{vmatrix}$

$\Rightarrow \Delta=(a\,\alpha^3+b\,\alpha^2+c\,\alpha+d)(bd-c^2)$

$\therefore \Delta=0$

$\Rightarrow$ either b, c, d are in G.P. or α is a root of

$ax^3+bx^3+cx+d=0$

38 **(a)**

We have,

$\begin{vmatrix} \cos C & \tan A & 0 \\ \sin B & 0 & -\tan A \\ 0 & \sin B & \cos C \end{vmatrix}$

$=\frac{1}{\cos^2 A}\begin{vmatrix} \cos C\cos A & \sin A & 0 \\ \sin B\cos A & 0 & -\sin A \\ 0 & \sin B & \cos C \end{vmatrix}$

$\begin{bmatrix} \text{Applying } R_1 \to R_1\cos A \\ R_2 \to R_2\cos A \end{bmatrix}$

$=\frac{1}{\cos A}\begin{vmatrix} \cos C & \sin A & 0 \\ \sin B & 0 & -\sin A \\ 0 & \sin B & \cos C \end{vmatrix}$

$=\frac{1}{\cos A}\{\sin A\sin B\cos C-\sin A\sin B\cos C\}$

$=0$

39 **(b)**

Applying $C_3 \to C_3-C_1$,we get

$\Delta=\begin{vmatrix} 1 & \alpha & \alpha^2-1 \\ \cos(p-d)a & \cos pa & 0 \\ \sin(p-d)a & \sin pa & 0 \end{vmatrix}$

$=(\alpha^2-1)\{-\cos pa\,\sin(p-d)a$
$+\sin pa\cos(p-d)a\}$

$=(\alpha^2-1)\sin\{-(p-d)a+pa\}$

$\Rightarrow \Delta=(\alpha^2-1)\sin da$

Which is independent of p.

40 **(c)**

Let $\Delta=\begin{vmatrix} a & b\omega^2 & a\omega \\ b\omega & c & b\omega^2 \\ c\omega^2 & a\omega & c \end{vmatrix}$

Applying $C_3 \to C_3-\omega C_1$

$=\begin{vmatrix} a & b\omega^2 & 0 \\ b\omega & c & 0 \\ c\omega^2 & a\omega & 0 \end{vmatrix}$

$=0$

42 **(b)**

We have, $\Delta_1 = \begin{vmatrix} x & b & b \\ a & x & b \\ a & a & x \end{vmatrix} = x^3 - 3abx + ab^2 + a^2b$

$\Rightarrow \frac{d}{dx}\Delta_1 = 3(x^2 - ab)$ and $\Delta_2 = \begin{vmatrix} x & b \\ a & x \end{vmatrix} = x^2 - ab$

$\therefore \frac{d}{dx}(\Delta_1) = 3(x^2 - ab) = 3\Delta_2$

43 **(b)**

Given $f(\theta) = \begin{vmatrix} \cos^2\theta & \cos\theta\sin\theta & -\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta & \cos\theta \\ \sin\theta & -\cos\theta & 0 \end{vmatrix}$

$= \cos^2\theta(0 + \cos^2\theta) - \cos\theta\sin\theta(0 - \sin\theta\cos\theta)$
$- \sin\theta(-\cos^2\theta\sin\theta - \sin^3\theta)$
$= \cos^4\theta + 2\sin^2\theta\cos^2\theta + \sin^4\theta$
$= \cos^4\theta + \sin^2\theta\cos^2\theta + \sin^2\theta$
$= \cos^2\theta(\cos^2\theta + \sin^2\theta) + \sin^2\theta = 1$

$\therefore$ For all, $\theta, f(\theta) = 1$

44 **(d)**

Given that $C = 2\cos\theta$

and $\Delta = \begin{vmatrix} C & 1 & 0 \\ 1 & C & 1 \\ 6 & 1 & C \end{vmatrix} = C(C^2 - 1) - 1(C - 6)$

$\Delta = 2\cos\theta(4\cos^2\theta - 1) - (2\cos\theta - 6)$ $(\because C = 2\cos\theta)$

$\Rightarrow \Delta = 8\cos^3\theta - 4\cos\theta + 6$

45 **(d)**

We have,

$f(x) = \begin{vmatrix} \sin x & \cos x & \tan x \\ x^3 & x^2 & x \\ 2x & 1 & 1 \end{vmatrix}$

$\Rightarrow \frac{f(x)}{x} = \begin{vmatrix} \frac{\sin x}{x} & \cos x & \tan x \\ x^2 & x^2 & x^2 \\ 2 & 1 & 1 \end{vmatrix}$

$\Rightarrow \frac{f(x)}{x^2} = \begin{vmatrix} \frac{\sin x}{x} & \cos x & \tan x \\ x & x & 1 \\ 2 & 1 & 1 \end{vmatrix}$

$\Rightarrow \lim_{x\to 0} \frac{f(x)}{x^2} = \begin{vmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 2 & 1 & 1 \end{vmatrix} = -1(1-2) = 1$

46 **(a)**

$\begin{vmatrix} [e] & [\pi] & [\pi^2 - 6] \\ [\pi] & [\pi^2 - 6] & [e] \\ [\pi^2 - 6] & [e] & [\pi] \end{vmatrix}$

$= \begin{vmatrix} 2 & 3 & 3 \\ 3 & 3 & 2 \\ 3 & 2 & 3 \end{vmatrix}$

$= 2(9-4) - 3(9-6) + 3(6-9)$
$= 10 - 9 - 9 = -8$

47 **(b)**

$AB = \begin{bmatrix} 3 & 5 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} 1 & 17 \\ 0 & -10 \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 2 & 34 \end{bmatrix}$

$\Rightarrow |AB| = \begin{bmatrix} 3 & 1 \\ 2 & 34 \end{bmatrix} = 100$

48 **(c)**

Given that,

$\Delta = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ 3 & 9 & 13 \end{vmatrix}$

Applying $R_1 \to R_1 + 2R_3$

$\Delta = \begin{vmatrix} 7 & 20 & 29 \\ 2 & 5 & 7 \\ 3 & 9 & 13 \end{vmatrix}$

$\Rightarrow \Delta = \Delta'$

49 **(d)**

$\begin{vmatrix} 2xy & x^2 & y^2 \\ x^2 & y^2 & 2xy \\ y^2 & 2xy & x^2 \end{vmatrix}$

$= 2xy(x^2y^2 - 4x^2y^2)$
$- x^2(x^4 - 2xy^3) + y^2(2x^3y - y^4)$
$= -6x^3y^3 - x^6 + 2x^3y^3 + 2x^3y^3 - y^6$
$= -(x^6 + y^6 + 2x^3y^3)$
$= -(x^3 + y^3)^2$

50 **(a)**

We have,

$\Delta = abc\left(1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right)$

$\therefore \Delta = 0 \Rightarrow \frac{1}{a} + \frac{1}{b} + \frac{1}{c} = -1$

51 **(c)**

$\begin{vmatrix} b^2c^2 & bc & b+c \\ c^2a^2 & ca & c+a \\ a^2b^2 & ab & a+b \end{vmatrix}$

On multiplying R_1, R_2, R_3 by a, b, c respectively and divide the whole by abc

$= \frac{1}{abc}\begin{vmatrix} ab^2c^2 & abc & a(b+c) \\ bc^2a^2 & bca & b(c+a) \\ a^2b^2c & abc & c(a+b) \end{vmatrix}$

On taking common abc from C_1 and C_2, we get

$= \frac{(abc)(abc)}{abc}\begin{vmatrix} bc & 1 & ab+ac \\ ca & 1 & bc+ab \\ ab & 1 & ca+bc \end{vmatrix}$

Now, $C_1 \to C_1 + C_3$

$= abc\begin{vmatrix} ab+bc+ca & 1 & ab+ac \\ ca+bc+ab & 1 & bc+ab \\ ab+bc+ca & 1 & ca+bc \end{vmatrix}$

$= (abc)(ab+bc+ca)\begin{vmatrix} 1 & 1 & ab+ac \\ 1 & 1 & bc+ab \\ 1 & 1 & ca+bc \end{vmatrix}$

$= 0$ $[\because$ two columns are identical$]$

52 **(b)**

We have, $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = \lambda$

Applying $C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_1$

$\Rightarrow \begin{vmatrix} 1+a & -a & -a \\ 1 & b & 0 \\ 1 & 0 & c \end{vmatrix} = \lambda$

On expanding w.r.t. R_3, we get

$ab + bc + ca + abc = \lambda$ ...(i)

Given $a^{-1} + b^{-1} + c^{-1} = 0$

$\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 0$

$\Rightarrow ab + bc + ca = 0$

From Eq. (i), $\lambda = abc$

53 **(c)**

We have,

$\begin{vmatrix} x+1 & x+2 & x+a \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix}$

$= \frac{1}{2}\begin{vmatrix} x+1 & x+2 & x+a \\ 2x+4 & 2x+6 & 2x+2b \\ x+2 & x+4 & x+c \end{vmatrix}$ [Applying $R_2 \to 2R_2$]

$= \frac{1}{2}\begin{vmatrix} x+1 & x+2 & x+a \\ 0 & 0 & 0 \\ x+2 & x+4 & x+c \end{vmatrix}$ [Applying $R_2 - (R_1 + R_3)$]

$= 0$

54 **(b)**

$\begin{vmatrix} a-b & b-c & c-a \\ x-y & y-z & z-x \\ p-q & p-r & r-p \end{vmatrix} = \begin{vmatrix} 0 & b-c & c-a \\ 0 & y-z & z-x \\ 0 & q-r & r-p \end{vmatrix} = 0$

$(C_1 \to C_1 + C_2 + C_3)$

55 **(c)**

$\begin{vmatrix} a-b+c & -a-b+c & 1 \\ a+b+2c & -a+b+2c & 2 \\ 3c & 3c & 3 \end{vmatrix}$

$\begin{vmatrix} 2a & -2a & 0 \\ a+b+2c & -a+b+2c & 2 \\ 3c & 3c & 3 \end{vmatrix}$

[using $R_1 \to R_1 + R_2 - R_3$]

$= 2a(-3a + 3b + 6c - 6c) + 2a(3a + 3b + 6c - 6c)$

$= 12ab$

56 **(a)**

Ratio of cofactor to its minor of the element -3, which is in the 3rd row and 2nd column $= (-1)^{3+2} = -1$

57 **(d)**

We have,

$\Delta = \begin{vmatrix} x+1 & \omega & \omega^2 \\ \omega & x+\omega^2 & 1 \\ \omega^2 & 1 & x+\omega \end{vmatrix}$

$\Rightarrow \Delta$

$= \begin{vmatrix} x+1+\omega+\omega^2 & x+\omega+\omega^2+1 & x+1+\omega+ \\ \omega & x+\omega^2 & 1 \\ \omega^2 & 1 & x+\omega \end{vmatrix}$

[Applying $R_1 \to R_1 + R_2 + R_3$]

$\Rightarrow \Delta = (x+1+\omega+\omega^2)\begin{vmatrix} 1 & 1 & 1 \\ \omega & x+\omega^2 & 1 \\ \omega^2 & 1 & x+\omega \end{vmatrix}$

$\Rightarrow \Delta = x\begin{vmatrix} 1 & 0 & 0 \\ \omega & x+\omega^2-\omega & 1-\omega \\ \omega^2 & 1-\omega^2 & x+\omega-\omega^2 \end{vmatrix}$

$\Rightarrow \Delta = x[(x+\omega^2-\omega)(x+\omega-\omega^2) - (1-\omega)(1-\omega^2)]$

$\therefore \Delta = 0 \Rightarrow x = 0$

59 **(d)**

Applying $C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_1$ to the given determinant and expanding it along first now, we get

$\Rightarrow (\sin B - \sin A)(\sin C - \sin A)$

$\times \begin{vmatrix} 1 & 1 \\ 1+\sin B+\sin A & 1+\sin C+\sin A \end{vmatrix} = 0$

$\Rightarrow (\sin B - \sin A)(\sin C - \sin A)(\sin C - \sin B) = 0$

$\Rightarrow \sin B = \sin A$ or $\sin C = \sin A$ or $\sin C = \sin B$

$\Rightarrow A = B$ or $B = C$ or $C = A$

$\Rightarrow \Delta ABC$ is isosceles

60 **(c)**

We have, $D_r = \begin{vmatrix} 2^{r-1} & 3^{r-1} & 4^{r-1} \\ x & y & z \\ 2^n - 1 & (3^n-1)/2 & (4^n-1)/3 \end{vmatrix}$

$\Rightarrow \sum_{r=1}^{n} D_r = \begin{vmatrix} \sum_{r=1}^{n} 2^{r-1} & \sum_{r=1}^{n} 3^{r-1} & \sum_{r=1}^{n} 4^{r-1} \\ x & y & z \\ 2^n - 1 & (3^n-1)/2 & (4^n-1)/3 \end{vmatrix}$

$\Rightarrow \sum_{r=1}^{n} D_r = \begin{vmatrix} 2^n - 1 & (3^n-1)/2 & (4^n-1)/3 \\ x & y & z \\ 2^n - 1 & (3^n-1)/2 & (4^n-1)/3 \end{vmatrix}$

$\sum_{r=1}^{n} D_r = 0$ ($\because$ two rows are same)

61 **(b)**

We have,

$\begin{vmatrix} 1 & 1 & 1 \\ 1+\sin A & 1+\sin B & 1+\sin C \\ \sin A+\sin^2 A & \sin B+\sin^2 B & \sin C+\sin^2 C \end{vmatrix} = 0$

$\Rightarrow \begin{vmatrix} 1 & 1 & 1 \\ \sin A & \sin B & \sin C \\ \sin A+\sin^2 A & \sin B+\sin^2 B & \sin C+\sin^2 C \end{vmatrix} = 0$

Applying $R_2 \to R_2 - R_1$

$\Rightarrow \begin{vmatrix} 1 & 1 & 1 \\ \sin A & \sin B & \sin C \\ \sin^2 A & \sin^2 B & \sin^2 C \end{vmatrix} = 0$ Applying $R_3 \to R_3 - R_2$

$\Rightarrow (\sin A - \sin B)(\sin B - \sin C)(\sin C - \sin A) = 0$

$\Rightarrow \sin A = \sin B$ or, $\sin B = \sin C$ or, $\sin C = \sin A$

$\Rightarrow \Delta ABC$ is isosceles

62 **(c)**

We have,

$$\det(A)=\begin{vmatrix}1 & \sin\theta & 1\\ -\sin\theta & 1 & \sin\theta\\ -1 & -\sin\theta & 1\end{vmatrix}$$
$$=2(1+\sin^2\theta)$$

Now,

$0\le\sin^2\theta\le 1$ for all $\theta\in[0,2\pi)$

$\Rightarrow 2\le 2+2\sin^2\theta\le 4$ for all $\theta\in[0,2\pi)$

$\Rightarrow \text{Det}(A)\in[2,4]$

63 **(d)**

Let $\Delta=\begin{vmatrix}1 & 5 & \pi\\ \log_e e & 5 & \sqrt{5}\\ \log_{10}10 & 5 & e\end{vmatrix}$

$\Rightarrow \Delta=\begin{vmatrix}1 & 5 & \pi\\ 1 & 5 & \sqrt{5}\\ 1 & 5 & e\end{vmatrix}=5\begin{vmatrix}1 & 1 & \pi\\ 1 & 1 & \sqrt{5}\\ 1 & 1 & e\end{vmatrix}$ $(\because \log_a a=1)$

$=0$ $(\because$ two columns are identical)

64 **(a)**

Applying $C_1\to C_1+C_2+C_3$, we get

$f(x)$

$$=\begin{vmatrix}1+a^2x+x+xb^2+x+c^2x & (1+b^2)x & (1+c^2)x\\ x+a^2x+1+b^2x+x+c^2x & (1+b^2x) & (1+c^2)x\\ x+a^2x+x+b^2x+1+c^2x & (1+b^2)x & (1+c^2x)\end{vmatrix}$$

$$=\begin{vmatrix}1 & (1+b^2)x & (1+c^2)x\\ 1 & 1+b^2x & (1+c^2)x\\ 1 & (1+b^2)x & 1+c^2x\end{vmatrix}$$

$[\because a^2+b^2+c^2+2=0]$

Applying $R_1\to R_1-R_3,\ R_2\to R_2-R_3$

$$=\begin{vmatrix}0 & 0 & x-1\\ 0 & 1-x & x-1\\ 1 & (1+b^2)x & 1+c^2x\end{vmatrix}$$
$$=1[0-(x-1)(1-x)]$$

$=(x-1)^2$

$\Rightarrow f(x)$ is a polynomial of degree 2

65 **(c)**

Since system of equations is consistent.

$\therefore \begin{vmatrix}1 & 1 & -1\\ 2 & -1 & -c\\ -b & 3b & -c\end{vmatrix}=0$

$\Rightarrow c+bc-6b+b+2c+3bc=0$

$\Rightarrow 3c+4bc-5b=0$

$\Rightarrow c=\dfrac{5}{3+4b}$

But $c<1\Rightarrow \frac{5b}{3+4b}<1$

$\Rightarrow \dfrac{b-3}{3+4b}<0$

$\Rightarrow b\in\left(-\dfrac{3}{4},3\right)$

66 **(a)**

Applying $R_2\to R_2-R_3$, we get

$$\begin{vmatrix}1 & 1 & 1\\ 4 & 4 & 4\\ (2^x-2^{-x})^2 & (3^x-3^{-x})^2 & (5^x-5^{-x})^2\end{vmatrix}$$

$$=4\begin{vmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ (2^x-2^{-x})^2 & (3^x-3^{-x})^2 & (5^x-5^{-x})^2\end{vmatrix}$$

$=4\times 0=0$ $[\therefore$ two rows are identical]

67 **(b)**

We have,

$AA^{-1}=I$

$\Rightarrow \det(AA^{-1})=\det(I)$

$\Rightarrow \det(A)\det(A^{-1})=1$

$\left[\begin{matrix}\because \det(A\,B)=\det(A)\det(B)\\ \text{and}, \det(I)=1\end{matrix}\right]$

$\Rightarrow \det(A^{-1})=\dfrac{1}{\det(A)}$

68 **(b)**

We have,

$$\begin{vmatrix}1+ax & 1+bx & 1+cx\\ 1+a_1x & 1+b_1x & 1+c_1x\\ 1+a_2x & 1+b_2x & 1+c_2x\end{vmatrix}$$

$$=\begin{vmatrix}1+ax & (b-a)x & (c-a)x\\ 1+a_1x & (b_1-a_1)x & (c_1-a_1)x\\ 1+a_2x & (b_2-a_2)x & (c_2-a_2)x\end{vmatrix}$$

Applying $C_2\to C_2-C_1$ and $C_3\to C_3-C_1$

$$=x^2\begin{vmatrix}1+ax & b-a & c-a\\ 1+a_1x & b_1-a_1 & c_1-a_1\\ 1+a_2x & b_2-a_2 & c_2-a_2\end{vmatrix}$$

$$=x^2[(1+ax)\{(b_1-a_1)(c_2-a_2)-(b_2-a_2)(c_1-a_1)\}-(1+a_1x)\{(b-a)(c_2-a_2)-(c-a)(b_2-a_2)\}+(1+a_2x)\{(b-a)(c_1-a_1)-(c-a)(b_1-a_1)\}]$$

$=x^2(\lambda\,x+\mu)$, where λ and μ are constants

$=\mu\,x^2+\lambda\,x^3$

Hence, $A_0=A_1=0$

69 **(b)**

$f(x)=\begin{vmatrix}a & -1 & 0\\ ax & a & -1\\ ax^2 & ax & a\end{vmatrix}$

Applying $R_3\to R_3-xR_2$

$f(x)=\begin{vmatrix}a & -1 & 0\\ ax & a & -1\\ 0 & 0 & a+x\end{vmatrix}=(a+x)(a^2+ax)$

$\Rightarrow f(x)=a(a+x)^2$

$\therefore f(2x)=a(a+2x)^2$

$\Rightarrow f(2x)-f(x)=ax(2a+3x)$

70 **(c)**

$\begin{vmatrix}-12 & 0 & \lambda\\ 0 & 2 & -1\\ 2 & 1 & 15\end{vmatrix}$=-360

$\Rightarrow\ -12(30+1)-4\lambda=-360$

$\Rightarrow \quad -372+360=4\lambda \;\Rightarrow\; \lambda=-\frac{12}{4}=-3$

71 **(c)**

Let $A=\begin{vmatrix}1 & \omega & \omega^2\\ \omega & \omega^2 & 1\\ \omega^2 & 1 & \omega\end{vmatrix}=$

$\begin{vmatrix}1+\omega+\omega^2 & \omega & \omega^2\\ 1+\omega+\omega^2 & \omega^2 & 1\\ 1+\omega+\omega^2 & 1 & \omega\end{vmatrix}$

$[C_1 \to C_1+C_2+C_3$

$=\begin{vmatrix}0 & \omega & \omega^2\\ 0 & \omega^2 & 1\\ o & 1 & \omega\end{vmatrix}=0 \qquad [\because 1+\omega+\omega^2=0]$

72 **(b)**

Applying $C_1 \to C_1+C_2$, we get

$\begin{vmatrix}{}^{10}C_4+{}^{10}C_5 & {}^{10}C_5 & {}^{11}C_m\\ {}^{11}C_6+{}^{11}C_7 & {}^{11}C_7 & {}^{12}C_{m+2}\\ {}^{12}C_8+{}^{12}C_9 & {}^{12}C_9 & {}^{13}C_{m+4}\end{vmatrix}=0$

$\Rightarrow \begin{vmatrix}{}^{11}C_5 & {}^{10}C_5 & {}^{11}C_m\\ {}^{12}C_7 & {}^{11}C_7 & {}^{12}C_{m+2}\\ {}^{13}C_9 & {}^{12}C_9 & {}^{13}C_{m+4}\end{vmatrix}=0$

It means either two rows or two columns are identical.

$\therefore {}^{11}C_5={}^{11}C_m,\ {}^{12}C_7={}^{12}C_{m+2},\ {}^{13}C_9={}^{13}C_{m+4}$

$\Rightarrow \quad m=5$

73 **(b)**

Given, $\begin{vmatrix}1 & 1 & 0\\ 2 & 0 & 3\\ 5 & -6 & x\end{vmatrix}=29$

$\Rightarrow 1(0+18)-1(2x-15)=29$

$\Rightarrow 2x=4 \Rightarrow x=2$

74 **(a)**

Applying $C_1 \to C_1+C_2$, we get

$\begin{vmatrix}\sin^2 x & \cos^2 x & 1\\ \cos^2 x & \sin^2 x & 1\\ -10 & 12 & 2\end{vmatrix}=\begin{vmatrix}1 & \cos^2 x & 1\\ 1 & \sin^2 x & 1\\ 2 & 12 & 2\end{vmatrix}=0$

75 **(b)**

Since, $|A|=-1, |B|=3$

$\therefore |AB|=|A||B|=-3$

Now, $|3AB|=(3)^3(-3)=-81$

77 **(d)**

Applying $C_3 \to C_3-\alpha C_1+C_2$ to the given determinant, we get

$\begin{vmatrix}a & b & 0\\ b & c & 0\\ 2 & 1 & -2\alpha+1\end{vmatrix}=(1-2\alpha)(ac-b^2)$

So, if the determinant is zero, we must have

$(1-2\alpha)(ac-b^2)=0$

$\Rightarrow \; 1-2\alpha=0$

or $(ac-b^2)=0$

$\Rightarrow \; \alpha=\frac{1}{2}$ or $ac=b^2$

Which means a, b, c are in GP.

78 **(a)**

We have, $\begin{vmatrix}x & 3 & 7\\ 2 & x & 2\\ 7 & 6 & x\end{vmatrix}=0$

$\Rightarrow (x+9)\begin{vmatrix}1 & 1 & 1\\ 2 & x & 2\\ 7 & 6 & x\end{vmatrix}=0 \;\; (R_1 \to R_1+R_2+R_3)$

$\Rightarrow (x+9)\{1(x^2-12)-1(2x-14)$
$+1(12-7x)\}=0$

$\Rightarrow (x+9)(x^2-9x+14)=0$

$\Rightarrow \; (x+9)(x-2)(x-7)=0$

$\therefore$ The other two roots are 2 and 7.

79 **(a)**

Let $A \equiv \begin{vmatrix}a-x & c & b\\ c & b-x & a\\ b & a & c-x\end{vmatrix}=0$

Applying $C_1 \to C_1+C_2+C_3$

$=\begin{vmatrix}a+b+c-x & c & b\\ a+b+c-x & b-x & a\\ a+b+c-x & a & c-x\end{vmatrix}$

$=(a+b+c-x)\begin{vmatrix}1 & c & b\\ 1 & b-x & a\\ 1 & a & c-x\end{vmatrix}$

$\Rightarrow \; (a+b+c-x)[1\{(b-x)(c-x)-a^2\}$
$-c(c-x-a)+b(a-b+x)]$
$=0$

$\Rightarrow \; (a+b+c-x)[bc-bx-cx+x^2-a^2-c^2$
$+xc+ac+ab-b^2+bx]=0$

$\Rightarrow \; (a+b+c-x)[x^2-(a^2+b^2+c^2)+ab+bc$
$+ca]=0$

$\because \; ab+bc+ca=0$ (given)

$\Rightarrow$ either $x=a+b+c$ or $x=(a^2+b^2+c^2)^{1/2}$

80 **(b)**

We have,

$\begin{vmatrix}x-1 & 1 & 1\\ 1 & x-1 & 1\\ 1 & 1 & x-1\end{vmatrix}=0$

$\Rightarrow \begin{vmatrix}x+1 & 1 & 1\\ x+1 & x-1 & 1\\ x+1 & 1 & x-1\end{vmatrix}=0$ [Applying $C_1 \to C_1+C_2+C_3$]

$\Rightarrow (x+1)\begin{vmatrix}1 & 1 & 1\\ 1 & x-1 & 1\\ 1 & 1 & x-1\end{vmatrix}=0$

$\Rightarrow (x+1)\begin{vmatrix}1 & 1 & 1\\ 0 & x-2 & 0\\ 0 & 0 & x-2\end{vmatrix}=0$

$\begin{bmatrix}\text{Applying } R_2 \to R_2-R_1,\\ R_3 \to R_3-R_1\end{bmatrix}$

$\Rightarrow (x+1)(x-2)^2=0$

$\Rightarrow x=-1, 2$

81 **(c)**

Let $A=\begin{vmatrix}1 & 2 & 3\\ 1^3 & 2^3 & 3^3\\ 1^5 & 2^5 & 3^5\end{vmatrix}=1.2.3\begin{vmatrix}1 & 2 & 3\\ 1^2 & 2^2 & 3^2\\ 1^4 & 2^4 & 3^4\end{vmatrix}$

$= 6\begin{vmatrix}1 & 1 & 1\\1 & 4 & 9\\1 & 16 & 81\end{vmatrix} = 6\begin{vmatrix}1 & 0 & 0\\1 & 3 & 5\\1 & 15 & 65\end{vmatrix}$

$[C_2 \to C_2 - C_1, C_3 \to C_3 - C_2]$

$= 6.3.5\begin{vmatrix}1 & 0 & 0\\1 & 1 & 1\\1 & 5 & 13\end{vmatrix} = 90[1(13-5)] = 720 = 6!$

82 **(c)**

$\because\ |A^3| = |A|^3 = 125$

$\Rightarrow \begin{bmatrix}\alpha & 2\\2 & \alpha\end{bmatrix} = 5$

$\Rightarrow \alpha^2 - 4 = 5 \quad \Rightarrow \quad \alpha = \pm 3$

84 **(b)**

Given, angles of a triangle are A, B and C. We know that $A + B + C = \pi$, therefore

$A + B = \pi - C$

$\Rightarrow \cos(A+B) = \cos(\pi - C) = -\cos C$

$\Rightarrow \cos A \cos B - \sin A \sin B = -\cos C$

$\Rightarrow \cos A \cos B + \cos C = \sin A \sin B \quad$ ...(i)

Let $\Delta = \begin{vmatrix}-1 & \cos C & \cos B\\\cos C & -1 & \cos A\\\cos B & \cos A & -1\end{vmatrix}$

$= -(1 - \cos^2 A)$
$+ \cos C(\cos C$
$+ \cos A \cos B)$
$+ \cos B(\cos B + \cos A \cos C)$

$= -\sin^2 A + \cos C(\sin A \sin B) +$
$\cos B(\sin A \sin C)$ [from Eq.(i)]

$= -\sin^2 A + \sin A(\sin B \cos C + \cos B \sin C)$

$= -\sin^2 A + \sin A \sin(B + C)$

$= -\sin^2 A + \sin^2 A = 0$ $[\because \sin(B+C) = \sin(\pi - A) = \sin A]$

85 **(a)**

We have,

$\Delta = \begin{vmatrix}a^2+x & ab & ac\\ab & b^3+x & bc\\ac & bc & c^2+x\end{vmatrix}$

$\Rightarrow \Delta = \frac{1}{abc}\begin{vmatrix}a^3+ax & a^2b & a^2c\\ab^2 & b^3+bx & b^2c\\ac^2 & bc^2 & c^3+cx\end{vmatrix}$

$\begin{bmatrix}\text{Applying } C_1(a),\\ C_2(b), C_3(c)\end{bmatrix}$

$\Rightarrow \Delta = \begin{vmatrix}a^2+x & a^2 & a^2\\b^2 & b^2+x & b^2\\c^2 & c^2 & c^2+x\end{vmatrix}$

$\Rightarrow \Delta = (a^2+b^2+c^2+x)\begin{vmatrix}1 & 1 & 1\\b^2 & b^2+x & b^2\\c^2 & c^2 & c^2+x\end{vmatrix}$

[Applying $R_1 \to R_1 + R_2 + R_3$]

$\Rightarrow \Delta = (a^2+b^2+c^2+x)\{(b^2x + c^2x + x^2) - (b^2x) + (-c^2x)\}$

$\Rightarrow \Delta = x^2(a^2+b^2+c^2+x)$

$\Rightarrow x^2$ is a factor Δ

86 **(a)**

Given that, $\begin{vmatrix}x+1 & x+2 & x+3\\x+2 & x+3 & x+4\\x+a & x+b & x+c\end{vmatrix} = 0$

$\Rightarrow \begin{vmatrix}-1 & -1 & x+3\\-1 & -1 & x+4\\a-b & b-c & x+c\end{vmatrix} = 0 \quad \begin{pmatrix}C_1 \to C_1 - C_2\\C_2 \to C_2 - C_3\end{pmatrix}$

$\Rightarrow \begin{vmatrix}0 & 0 & -1\\-1 & -1 & x+4\\a-b & b-c & x+c\end{vmatrix} = 0 \quad (R_1 \to R_1 - R_2)$

$\Rightarrow (-1)(-b + c + a - b) = 0$

$\Rightarrow 2b - a - c = 0$

$\Rightarrow a + c = 2b$

$\therefore a, b, c$ in AP.

87 **(b)**

Given, $A = \begin{vmatrix}1 & 0 & 0\\x & 1 & 0\\x & x & 1\end{vmatrix} \Rightarrow A = 1$

$\therefore A^3 - 4A^2 + 3A + I = (1)^3 - 4(1)^2 + 3(1) + I$
$= I$

88 **(a)**

Let $\Delta = \begin{vmatrix}1 & x & y\\2 & \sin x + 2x & \sin y + 3y\\3 & \cos x + 3x & \cos y + 3y\end{vmatrix}$

$= \begin{vmatrix}1 & x & y\\0 & \sin x & \sin y\\0 & \cos x & \cos y\end{vmatrix} \quad \begin{pmatrix}R_2 \to R_2 - 2R_1,\\R_3 \to R_3 - 3R_1\end{pmatrix}$

$= \sin x\ \cos y - \cos x\ \sin y = \sin(x - y)$

89 **(d)**

We have, $\Delta = \frac{1}{abc}\begin{vmatrix}a^3+ax & a^2b & a^2c\\ab^2 & b^3+bx & b^2c\\c^2a & c^2b & c^2+xc\end{vmatrix}$

Taking a, b, c common in columns Ist, IInd and IIIrd, we get,

$\Delta = \begin{vmatrix}a^2+x & a^2 & a^2\\b^2 & b^2+x & b^2\\c^2 & c^2 & c^2+x\end{vmatrix}$

Applying $R_1 \to R_1 + R_2 + R_3$

$= (a^2+b^2+c^2+x)\begin{vmatrix}1 & 1 & 1\\b^2 & b^2+x & b^2\\c^2 & c^2 & c^2+x\end{vmatrix}$

Applying $C_2 \to C_2 - C_1, C_3 \to C_3 - C_1$

$= (a^2+b^2+c^2+x)\begin{vmatrix}1 & 1 & 0\\b^2 & x & 0\\c^2 & 0 & x\end{vmatrix}$

$= x(x - b^2)(a^2+b^2+c^2+x)$

Hence, option (d) is correct.

90 **(a)**

Given, $\begin{vmatrix}bc & ca & ab\\ca & ab & bc\\ab & bc & ca\end{vmatrix} = 0$

$\Rightarrow (ab)^3 + (bc)^3 + (ca)^3 - 3a^2b^2c^2 = 0$

$\Rightarrow (ab + bc + ca)(a^2b^2 + b^2c^2 + c^2a^2 - ab^2c$
$-bc^2a - ca^2b) = 0$

$\Rightarrow \quad ab + bc + ca = 0$

$\Rightarrow \frac{1}{a}+\frac{1}{b}+\frac{1}{c}=0$

91 **(c)**

Given, $f(x) =$

$$\begin{vmatrix} 1 & 2(x-1) & 3(x-1)(x-2) \\ x-1 & (x-1)(x-2) & (x-1)(x-2)(x-3) \\ x & x(x-1) & x(x-1)(x-2) \end{vmatrix}$$

$= (x-1)(x-1)(x-2)\begin{vmatrix} 1 & 2 & 3 \\ x-1 & x-2 & x-3 \\ x & x & x \end{vmatrix}$

Applying $C_1 \to C_1 - C_2, C_2 \to C_2 - C_3$

$= (x-1)^2(x-2)\begin{vmatrix} -1 & -1 & 3 \\ 1 & 1 & x-3 \\ 0 & 0 & x \end{vmatrix}$

$= (x-1)^2(x-2)x(-1+1) = 0$

$\Rightarrow f(x) = 0$

$\therefore f(49) = 0$

92 **(b)**

Given that, $\begin{vmatrix} 1+ax & 1+bx & 1+cx \\ 1+a_1x & 1+b_1x & 1+c_1x \\ 1+a_2x & 1+b_2x & 1+c_2x \end{vmatrix}$

$= A_0 + A_1x + A_2x^2 + A_3x^3$

On putting $x = 0$ on both sides, we get

$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix} = A_0$

$\Rightarrow A_0 = 0$

94 **(d)**

We have,

$\begin{vmatrix} \cos\alpha & \sin\alpha & 0 \\ \cos\beta & \sin\beta & 0 \\ \cos\gamma & \sin\gamma & 0 \end{vmatrix}\begin{vmatrix} \cos\alpha & \sin\alpha & 0 \\ \cos\beta & \sin\beta & 0 \\ \cos\gamma & \sin\gamma & 0 \end{vmatrix}$

$= \begin{vmatrix} 1 & \cos(\beta-\alpha) & \cos(\gamma-\alpha) \\ \cos(\alpha-\beta) & 1 & \cos(\gamma-\beta) \\ \cos(\alpha-\gamma) & \cos(\beta-\gamma) & 1 \end{vmatrix}$

$\therefore \begin{vmatrix} 1 & \cos(\beta-\alpha) & \cos(\gamma-\alpha) \\ \cos(\alpha+\beta) & 1 & \cos(\gamma-\beta) \\ \cos(\alpha-\gamma) & \cos(\beta-\gamma) & 1 \end{vmatrix} = 0$

95 **(a)**

Given, $\begin{vmatrix} x & 3 & 7 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix} = 0$

$\Rightarrow x(x^2-12) - 3(2x-14) + 7(12-7x) = 0$

$\Rightarrow x^3 - 67x + 126 = 0$

$\Rightarrow (x+9)(x^2-9x+14) = 0$

$\Rightarrow (x+9)(x-2)(x-7) = 0$

$\Rightarrow x = -9, 2, 7$

Hence, the other two roots are 2, 7

96 **(c)**

From the sine rule, we have

$\Rightarrow \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} = k(\text{say}),$

$\Rightarrow \sin A = ak, \sin B = bk$ and $\sin C = ck$

$\therefore \begin{vmatrix} a^2 & b\sin A & c\sin A \\ b\sin A & 1 & \cos(B-C) \\ c\sin A & \cos(B-C) & 1 \end{vmatrix}$

$= \begin{vmatrix} a^2 & abk & ack \\ abk & 1 & \cos(B-C) \\ ack & \cos(B-C) & 1 \end{vmatrix}$

$= a^2\begin{vmatrix} 1 & \sin B & \sin C \\ \sin B & 1 & \cos(B-C) \\ \sin C & \cos(B-C) & 1 \end{vmatrix}$

$= a^2\begin{vmatrix} 1 & \sin(A+C) & \sin(A+B) \\ \sin(A+C) & 1 & \cos(B-C) \\ \sin(A+B) & \cos(B-C) & 1 \end{vmatrix}$

$= a^2\begin{vmatrix} \sin A & \cos A & 0 \\ \cos C & \sin C & 0 \\ \cos B & \sin B & 0 \end{vmatrix}\begin{vmatrix} \sin A & \cos A & 0 \\ \cos C & \sin C & 0 \\ \cos B & \sin B & 0 \end{vmatrix}$

$= a^2 \times 0 = 0$

97 **(b)**

Given, $D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+y \end{vmatrix}$

Applying $C_2 \to C_2 - C$ and $C_3 \to C_3 - C_1$

$= \begin{vmatrix} 1 & 0 & 0 \\ 1 & x & 0 \\ 1 & 0 & y \end{vmatrix} = xy$

Hence, D is divisible by both x and y.

98 **(a)**

Taking x common from $R_2$and $x(x-1)$common from R_3,we get

$f(x) = x^2(x-1)\begin{vmatrix} 1 & x & (x+1) \\ 2 & (x-1) & (x+1) \\ 3 & (x-2) & (x+1) \end{vmatrix}$

$\Rightarrow f(x) = x^2(x-1)(x+1)\begin{vmatrix} 1 & x & 1 \\ 2 & x-1 & 1 \\ 3 & x-2 & 1 \end{vmatrix}$

$= x^2(x^2-1)\begin{vmatrix} 1 & x & 1 \\ 1 & -1 & 0 \\ 2 & -2 & 0 \end{vmatrix} \quad \begin{bmatrix} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{bmatrix}$

$\Rightarrow f(x) = x^2(x^2-1)(-2+2) = 0$

$\Rightarrow \quad f(x) = 0$for all x

$\therefore \quad f(11) = 0$

99 **(b)**

Applying $R_2 \to R_2 - R_1, R_3 \to R_3 - R_1$,we get

$\begin{vmatrix} 1 & 4 & 20 \\ 0 & -6 & -15 \\ 0 & 2x-4 & 5x^2-20 \end{vmatrix} = 0$

$\Rightarrow 1[-6(5x^2-20) + 15(2x-4)] = 0$

$\Rightarrow x^2 - x - 2 = 0$

$\Rightarrow (x-2)(x+1) = 0$

$\Rightarrow \quad x = -1, 2$

100 **(b)**

We have,

$\begin{vmatrix} 3x-8 & 3 & 3 \\ 3 & 3x-8 & 3 \\ 3 & 3 & 3x-8 \end{vmatrix} = 0$

$$\Rightarrow \begin{vmatrix} 3x-2 & 3 & 3 \\ 3x-2 & 3x-8 & 3 \\ 3x-2 & 3 & 3x-8 \end{vmatrix} = 0$$ Applying $C_1 \to C_1 + C_2 + C_3$

$$\Rightarrow (3x-2)\begin{vmatrix} 1 & 3 & 3 \\ 1 & 3x-8 & 3 \\ 1 & 3 & 3x-8 \end{vmatrix} = 0$$

$$\Rightarrow (3x-2)\begin{vmatrix} 1 & 3 & 3 \\ 0 & 3x-11 & 0 \\ 0 & 0 & 3x-11 \end{vmatrix} = 0$$

Applying $R_2 \to R_2 - R_1$,

$R_3 \to R_3 - R_1$

$\Rightarrow (3x-2)(3x-11)^2 = 0$

$\Rightarrow x = 2/3, 11/3$

101 **(a)**

We have,

$$\begin{vmatrix} \alpha & x & x & x \\ x & \beta & x & x \\ x & x & \gamma & x \\ x & x & x & \delta \end{vmatrix}$$

$$= \begin{vmatrix} \alpha & x-\alpha & x-\alpha & x-\alpha \\ x & -(x-\beta) & 0 & 0 \\ x & 0 & -(x-\gamma) & 0 \\ x & 0 & 0 & -(x-\delta) \end{vmatrix}$$

$$= \alpha \begin{vmatrix} -(x-\beta) & 0 & 0 \\ 0 & -(x-\gamma) & 0 \\ 0 & 0 & -(x-\delta) \end{vmatrix} - x \begin{vmatrix} x-\alpha & x-\alpha & x-\alpha \\ 0 & -(x-\gamma) & 0 \\ 0 & 0 & -(x-\delta) \end{vmatrix}$$

$$+ x \begin{vmatrix} x-\alpha & x-\alpha & x-\alpha \\ -(x-\beta) & 0 & 0 \\ 0 & 0 & -(x-\delta) \end{vmatrix} - x \begin{vmatrix} x-\alpha & x-\alpha & x-\alpha \\ -(x-\beta) & 0 & 0 \\ 0 & -(x-\gamma) & 0 \end{vmatrix}$$

$= -\alpha(x-\beta)(x-\gamma)(x-\delta) - (x-\alpha)(x-\gamma)(x-\delta)$

$-x(x-\alpha)(x-\beta)(x-\delta) - x(x-\alpha)(x-\beta)(x-\gamma)$

$= -\alpha(x-\beta)(x-\gamma)(x-\delta) + x(x-\beta)(x-\gamma)(x-\delta)$

$-x(x-\beta)(x-\gamma)(x-\delta) - x(x-\alpha)(x-\gamma)(x-\delta)$

$-x(x-\alpha)(x-\beta)(x-\delta) - x(x-\alpha)(x-\beta)(x-\gamma)$

$= (x-\beta)(x-\gamma)(x-\delta)(x-\alpha)$
$- x[(x-\alpha)(x-\beta)(x-\gamma)$
$+ (x-\beta)(x-\gamma)(x-\delta)$
$+ (x-\gamma)(x-\delta)(x-\alpha) + (x-\alpha)(x-\beta)(x-\delta)]$

$= f(x) - xf'(x)$, where, $f(x) = (x-\alpha)(x-\beta)(x-\gamma)(x-\delta)$

102 **(b)**

Given $\begin{vmatrix} 1 & a & b \\ 1 & c & a \\ 1 & b & c \end{vmatrix} = 0$

$\Rightarrow c^2 - ab - a(c-a) + b(b-c) = 0$

$\Rightarrow a^2 + b^2 + c^2 - ab - bc - ca = 0$

$\Rightarrow \frac{1}{2}[(a-b)^2 + (b-c)^2 + (c-a)^2] = 0$

$\Rightarrow a = b = c$

So, ΔABC is equilateral triangle.

$\therefore \angle A = 60°, \angle B = 60°, \angle C = 60°$

$\sin^2 A + \sin^2 B + \sin^2 C$

$= \sin^2 60° + \sin^2 60° + \sin^2 60°$

$= \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2$

$= 3 \times \frac{3}{4} = \frac{9}{4}$

104 **(a)**

Given that, $\Delta = \begin{vmatrix} 1 & \omega^n & \omega^{2n} \\ \omega^{2n} & 1 & \omega^n \\ \omega^n & \omega^{2n} & 1 \end{vmatrix}$

Applying $C_1 \to C_1 + C_2 + C_3$

$$= \begin{vmatrix} 1+\omega^n+\omega^{2n} & \omega^n & \omega^{2n} \\ 1+\omega^n+\omega^{2n} & 1 & \omega^n \\ 1+\omega^n+\omega^{2n} & \omega^{2n} & 1 \end{vmatrix}$$

$$= \begin{vmatrix} 0 & \omega^n & \omega^{2n} \\ 0 & 1 & \omega^n \\ 0 & \omega^{2n} & 1 \end{vmatrix}$$

($\because$ If n multiple of 3, then $1 + \omega^n + \omega^{2n} = 0$)

$= 0$

105 **(b)**

$$\begin{vmatrix} x & 3 & 6 \\ 3 & 6 & x \\ 6 & x & 3 \end{vmatrix} = \begin{vmatrix} 2 & x & 7 \\ x & 7 & 2 \\ 7 & 2 & x \end{vmatrix} = \begin{bmatrix} 4 & 5 & x \\ 5 & x & 4 \\ x & 4 & 5 \end{bmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x+9 & x+9 & x+9 \\ 3 & 6 & x \\ 6 & x & 3 \end{vmatrix} = \begin{vmatrix} 9+x & x+9 & 9+x \\ x & 7 & 2 \\ 7 & 2 & x \end{vmatrix}$$

$$= \begin{bmatrix} 9+x & 9+x & 9+x \\ 5 & x & 4 \\ x & 4 & 5 \end{bmatrix} = 0$$

$$\Rightarrow (x+9)\begin{vmatrix} 1 & 0 & 0 \\ 3 & 3 & x-3 \\ 6 & x-6 & -3 \end{vmatrix} = (9+x)\begin{vmatrix} 1 & 0 & 0 \\ x & 7-x & 2-x \\ 7 & -5 & x-7 \end{vmatrix}$$

$$= (9+x)\begin{vmatrix} 1 & 0 & 0 \\ 5 & x-5 & -1 \\ x & 4-x & 5-x \end{vmatrix}$$

$= 0$

$\Rightarrow x + 9 = 0 \Rightarrow x = -9$

106 **(b)**

The given system of equations will have a unique solution, if

$$\begin{vmatrix} k & 2 & -1 \\ 0 & k-1 & -2 \\ 0 & 0 & k+2 \end{vmatrix} \neq 0 \Rightarrow k(k-1)(k+2) \neq 0$$

$$\Rightarrow k \neq 0,1,-2$$

108 **(d)**

$$\begin{vmatrix} a & b-y & c-z \\ a-x & b & c-z \\ a-x & b-y & c \end{vmatrix} = 0$$

Applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_2$

$$\Rightarrow \begin{vmatrix} a & b-y & c-z \\ -x & y & 0 \\ 0 & -y & z \end{vmatrix} = 0$$

$\Rightarrow a(yz) + x(bz - yz + cy - yz) = 0$

$\Rightarrow ayz + bzx + cyx = 2xyz$

$\Rightarrow \frac{a}{x} + \frac{b}{y} + \frac{c}{z} = 2$

109 **(a)**

Given,

$$\begin{vmatrix} 1 & \cos(\alpha-\beta) & \cos\alpha \\ \cos(\alpha-\beta) & 1 & \cos\beta \\ \cos\alpha & \cos\beta & 1 \end{vmatrix}$$ is symmetric determinant.

$\therefore$ Its value is

$1 + 2\cos(\alpha-\beta)\cos\alpha\cos\beta$

$-\cos^2\alpha - \cos^2\beta - \cos^2(\alpha-\beta)$

$= 1 - \cos^2\alpha - \cos^2\beta - \cos(\alpha-\beta)$

$[\cos(\alpha-\beta) - 2\cos\alpha\cos\beta]$

$= 1 - \cos^2\alpha - \cos^2\beta - \cos(\alpha-\beta)$

$[\cos(\alpha-\beta) - \cos(\alpha+\beta) - \cos(\alpha-\beta)]$

$= 1 - \cos^2\alpha - \cos^2\beta + \cos(\alpha-\beta)\cos(\alpha+\beta)$

$= 1 - \cos^2\alpha - \cos^2\beta$

$+ \cos^2\alpha\cos^2\beta - \sin^2\alpha\sin^2\beta$

$= 1 - \cos^2\alpha - \cos^2\beta(1 - \cos^2\alpha) - \sin^2\alpha\sin^2\beta$

$= (1 - \cos^2\alpha)(1 - \cos^2\beta) - \sin^2\alpha\sin^2\beta$

$= \sin^2\alpha\sin^2\beta - \sin^2\alpha\sin^2\beta = 0$

110 **(d)**

We have,

$$\Delta = \begin{vmatrix} 2\sin A\cos A & \sin C & \sin B \\ \sin C & 2\sin B\cos B & \sin A \\ \sin B & \sin A & 2\sin C\cos C \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} 2ka\cos A & kc & kb \\ kc & 2kb\cos B & ka \\ kb & ka & 2kc\cos C \end{vmatrix}$$ [Using: Sine rule]

$$\Rightarrow \Delta = k^3 \begin{vmatrix} 2a\cos A & c & b \\ c & 2b\cos B & a \\ b & a & 2c\cos C \end{vmatrix}$$

$\Rightarrow \Delta$

$$= k^3 \begin{vmatrix} a\cos A + a\cos A & a\cos B + b\cos A & c\cos A \\ a\cos B + b\cos A & b\cos B + b\cos B & b\cos C \\ c\cos A + a\cos C & b\cos C + c\cos B & c\cos C \end{vmatrix}$$

$$\Rightarrow \Delta = k^3 \begin{vmatrix} \cos A & a & 0 \\ \cos B & b & 0 \\ \cos C & c & 0 \end{vmatrix} \begin{vmatrix} a & \cos A & 0 \\ b & \cos B & 0 \\ c & \cos C & 0 \end{vmatrix}$$

$\Rightarrow \Delta = k^3 \times 0 \times 0 = 0$

111 **(d)**

Applying $R_1 \to R_1 + R_2 + R_3$ and taking common $(a+b+c)$ from R_1,we get

$$= (a+b+c)\begin{vmatrix} 1 & 1 & 1 \\ 2b & b-c-a & 0 \\ 2c & 0 & c-a-b \end{vmatrix}$$

Applying $C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_1$,

$$= (a+b+c)\begin{vmatrix} 1 & 0 & 0 \\ 2b & -b-c-a & -2b \\ 2c & 0 & -a-b-c \end{vmatrix}$$

$= (c+b+c)[(-b-c-a)(-a-b-c)]$

$= (a+b+c)^3$

112 **(b)**

We know that

$|AB| = |A||B|$

$\Rightarrow AB = 0$

$\Rightarrow |AB| = 0$

$\Rightarrow |A||B| = 0$

$\Rightarrow$ either $|A| = 0$ or, $|B| = 0$

113 **(a)**

The given system of equations will have a unique solution, if

$$\begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 3 & 2 & k \end{vmatrix} \neq 0 \Rightarrow k \neq 0$$

114 **(a)**

$\therefore a_1, a_2, \dots, a_n$ are in GP.

$\Rightarrow a_n, a_n + 2, a_{n+4}, \dots$ are also in GP.

Now, $(a_{n+2})^2 = a_n . a_{n+4}$

$\Rightarrow 2\log(a_{n+2}) = \log a_n + \log a_{n+4}$

Similarly, $2\log(a_{n+8}) = \log a_{n+6} + \log a_{n+10}$

$$\text{Now, } \Delta = \begin{vmatrix} \log a_n & \log a_{n+2} & \log a_{n+4} \\ \log a_{n+6} & \log a_{n+8} & \log a_{n+10} \\ \log a_{n+12} & \log a_{n+14} & \log a_{n+16} \end{vmatrix}$$

Applying $C_2 \to 2C_2 - C_1 - C_3$

$$\begin{vmatrix} \log a_n & 2\log a_{n+2} - \log a_n - \log a_{n+4} \\ \log a_{n+6} & 2\log a_{n+8} - \log a_{n+6} - \log a_{n+10} \\ \log a_{n+12} & 2\log a_{n+14} - \log a_{n+12} - \log a_{n+16} \end{vmatrix}$$

$$= \begin{vmatrix} \log a_n & 0 & \log a_{n+4} \\ \log a_{n+6} & 0 & \log a_{n+10} \\ \log a_{n+12} & 0 & \log a_{n+16} \end{vmatrix} = 0$$

116 **(a)**

We have,

$$\text{Coefficient of } x \text{ in } \begin{vmatrix} x & (1+\sin x)^3 & \cos x \\ 1 & \log(1+x) & 2 \\ x^2 & (1+x)^2 & 0 \end{vmatrix}$$

$=$ coefficient of x in

$$= \begin{vmatrix} x & \left(1+x-\frac{x^3}{3!}+\cdots\right)^3 & 1-\frac{x^2}{2!}+\cdots \\ 1 & x-\frac{x^2}{2}+\frac{x^3}{3}\cdots & 2 \\ x^2 & 1+2x+x^2 & 0 \end{vmatrix}$$

$= \text{Coefficient of } x \text{ in } \begin{vmatrix} x & 1 & 1 \\ 1 & x & 2 \\ x^2 & 1 & 0 \end{vmatrix}$

$= \text{Coefficient of } x \text{ in } [x(0-2)-(0-2x^2)+(1-x^3)] = -2$

119 **(d)**

On putting $x = 0$ in the given equation, we get

$g = \begin{vmatrix} 1 & 2 & 0 \\ 0 & 1 & 2 \\ 2 & 0 & 1 \end{vmatrix} = 9$

On differentiating given equation and then put $x = 0$, we get

$f = -5$

120 **(a)**

In ΔABC, given $\begin{vmatrix} 1 & a & b \\ 1 & c & a \\ 1 & b & c \end{vmatrix} = 0$

$\Rightarrow 1(c^2-ab)-a(c-a)+b(b-c)=0$

$\Rightarrow a^2+b^2+c^2-ab-bc-ca=0$

$\Rightarrow 2a^2+2b^2+2c^2-2ab-2bc-2ca=0$

$\Rightarrow (a^2+b^2-2ab)+(b^2+c^2-2bc) + (c^2+a^2-2ca)=0$

$\Rightarrow (a-b)^2+(b-c)^2+(c-a)^2=0$

Here, sum of squares of three numbers can be zero, if and only, if $a=b=c$.

$\Rightarrow \Delta ABC$ is an equilateral triangle.

$\Rightarrow \angle A = \angle B = \angle C = 60°$

$\therefore \sin^2 A + \sin^2 B + \sin^2 C = \sin^2 60° + \sin^2 60° + \sin^2 60°$

$= \left(\frac{3}{4}+\frac{3}{4}+\frac{3}{4}\right) = \frac{9}{4}$

122 **(d)**

$\Delta(-x)$

$$= \begin{vmatrix} f(-x)+f(x) & 0 & x^4 \\ 3 & f(-x)-f(x) & \cos x \\ x^4 & -2x & f(-x)f(x) \end{vmatrix}$$

$$\begin{vmatrix} f(x)+f(-x) & 0 & x^4 \\ 3 & f(x)-f(-x) & \cos x \\ x^4 & -2x & f(x)f(-x) \end{vmatrix}$$

$= -\Delta(x)$

So, $\Delta(x)$ is an odd function.

$\Rightarrow x^4\Delta(x)$ is an odd function

$\Rightarrow \int_{-2}^{2} x^4\Delta(x)dx = 0$

123 **(c)**

$$\begin{vmatrix} \cos(x-a) & \cos(x+a) & \cos x \\ \sin(x+a) & \sin(x-a) & \sin x \\ \cos a \tan x & \cos a \cot x & \operatorname{cosec} 2x \end{vmatrix}$$

$$\begin{vmatrix} \cos(x-a)+\cos(x-a) & \cos(x+a) & \cos x \\ \sin(x+a)+\sin(x-a) & \sin(x-a) & \sin x \\ \cos a\ (\tan x+\cot x) & \cos a \cot x & \operatorname{cosec} 2x \end{vmatrix}$$

$$= \begin{vmatrix} 2\cos x\cos a & \cos(x+a) & \cos x \\ 2\sin x\cos a & \sin(x-a) & \sin x \\ \cos a\left(\frac{\tan^2 x+1}{\tan x}\right) & \cos a \cot x & \operatorname{cosec} 2x \end{vmatrix}$$

$$= 2\cos a \begin{vmatrix} \cos x & \cos(x+a) & \cos x \\ \sin x & \sin(x-a) & \sin x \\ \operatorname{cosec} 2x & \cos a\cot x & \operatorname{cosec} 2x \end{vmatrix} = 0$$

[$\because$ two columns are identical]

125 **(a)**

Since $(x-k)$ will be common from each row which vanish by putting $x = k$. Therefore, $(x-k)^r$ will be a factor of $|A|$

126 **(d)**

Putting $x = 0$ in the given determinant equation we get

$a_0 = \begin{vmatrix} 0 & -1 & 3 \\ 1 & 2 & -3 \\ -3 & 4 & 0 \end{vmatrix}$

$= 1(0-9)+3(4+6)$

$= 30-9 = 21$

127 **(a)**

Given, $\begin{vmatrix} a & \cot\frac{A}{2} & \lambda \\ b & \cot\frac{B}{2} & \mu \\ c & \cot\frac{C}{2} & \gamma \end{vmatrix} = 0$

$\Rightarrow \begin{vmatrix} a & \frac{s(s-a)}{\Delta} & \lambda \\ b & \frac{s(s-b)}{\Delta} & \mu \\ c & \frac{s(s-c)}{\Delta} & \gamma \end{vmatrix} = 0$

$\left[\because \cot\frac{A}{2} = \frac{s(s-a)}{\sqrt{(s-a)(s-b)(s-c)}} = \frac{s(s-a)}{\Delta}\right]$

$\Rightarrow \frac{1}{r}\begin{vmatrix} a & s-a & \lambda \\ b & s-b & \mu \\ c & s-c & \gamma \end{vmatrix} = 0$ where $r = \frac{\Delta}{s}$

Applying $C_2 \to C_2 + C_1$

$\Rightarrow \frac{1}{r}\begin{vmatrix} a & s & \lambda \\ b & s & \mu \\ c & s & \gamma \end{vmatrix} = 0$

$\Rightarrow \frac{\Delta}{r^2}\begin{vmatrix} a & 1 & \lambda \\ b & 1 & \mu \\ c & 1 & \gamma \end{vmatrix} = 0$

Applying $R_1 \to R_1 - R_2, R_2 \to R_2 - R_3$

$\Rightarrow \frac{\Delta}{r^2}\begin{vmatrix} a-b & 0 & \lambda-\mu \\ b-c & 0 & \mu-\gamma \\ c & 1 & \gamma \end{vmatrix} = 0$

$\Rightarrow \frac{\Delta}{r^2}[(b-c)(\lambda-\mu)-(\mu-\gamma)(a-b)] = 0$

$\Rightarrow b(\lambda-\mu)-c(\lambda-\mu)-a(\mu-\gamma)+b(\mu-\gamma) = 0$

$\Rightarrow -a(\mu-\gamma)+b(\lambda-\mu+\mu-\gamma)-c(\lambda-\mu) = 0$

$\Rightarrow -a(\mu-\gamma)+b(\lambda-\gamma)-c(\lambda-\mu) = 0$

$\Rightarrow a(\mu-\gamma)+b(\gamma-\lambda)+c(\lambda-\mu) = 0$

129 **(d)**

Let $\Delta = \begin{vmatrix} x+2 & x+3 & x+a \\ x+4 & x+5 & x+b \\ x+6 & x+7 & x+c \end{vmatrix}$

Applying $C_2 \to C_2 - C_1$,we get

$\Delta = \begin{vmatrix} x+2 & 1 & x+a \\ x+4 & 1 & x+b \\ x+6 & 1 & x+c \end{vmatrix}$

Applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$\Rightarrow \Delta = \begin{vmatrix} x+2 & 1 & x+a \\ 2 & 0 & b-a \\ 4 & 0 & c-a \end{vmatrix}$

$= -1(2c-2a-4b+4a)$

$\Rightarrow \Delta = 2(2b-c-a)$ ...(i)

Since, a, b, c are in AP.

$\therefore b = \frac{a+c}{2}$

$\therefore \Delta = 2(a+c-c-a)$

$= 0$ [from Eq. (i)]

130 **(a)**

$\begin{vmatrix} \cos(A-P) & \cos(A-Q) & \cos(A-R) \\ \cos(B-P) & \cos(B-Q) & \cos(B-R) \\ \cos(C-P) & \cos(C-Q) & \cos(C-R) \end{vmatrix}$

$= \begin{vmatrix} \cos A\cos P+\sin A\sin P & \cos A\cos Q+\sin A\sin \\ \cos B\cos P+\sin B\sin P & \cos B\cos Q+\sin B\sin \\ \cos C\cos P+\sin C\sin P & \cos C\cos Q+\sin C\sin \end{vmatrix}$

The determinants can be rewritten as 8 determinants and the value of each of these 8 determinants is zero.

ie, $\cos P\cos Q\cos R\begin{vmatrix} \cos A & \cos A & \cos A \\ \cos B & \cos B & \cos B \\ \cos C & \cos C & \cos C \end{vmatrix} = 0$

Similarly, other determinants can be shown zero.

131 **(b)**

We have, $\Delta(x) = \begin{vmatrix} x^n & \sin x & \cos x \\ n! & \sin\frac{n\pi}{2} & \cos\frac{n\pi}{2} \\ a & a^2 & a^3 \end{vmatrix}$

$\frac{d^n}{dx^n}[\Delta(x)] = \begin{vmatrix} \frac{d^n}{dx^n}x^n & \frac{d^n}{dx^n}\sin x & \frac{d^n}{dx^n}\cos x \\ n! & \sin\left(\frac{n\pi}{2}\right) & \cos\left(\frac{n\pi}{2}\right) \\ a & a^2 & a^3 \end{vmatrix}$

($\because$ Differentiation of R_2 and R_3 are zero)

$\begin{vmatrix} n! & \sin\left(x+\frac{n\pi}{2}\right) & \cos\left(x+\frac{n\pi}{2}\right) \\ n! & \sin\left(\frac{n\pi}{2}\right) & \cos\left(\frac{n\pi}{2}\right) \\ a & a^2 & a^3 \end{vmatrix}$

$\Rightarrow [\Delta^n(x)]_{x=0}$

$= \begin{vmatrix} n! & \sin\left(0+\frac{n\pi}{2}\right) & \cos\left(0+\frac{n\pi}{2}\right) \\ n! & \sin\left(\frac{n\pi}{2}\right) & \cos\left(\frac{n\pi}{2}\right) \\ a & a^2 & a^3 \end{vmatrix}$

$= \begin{vmatrix} n! & \sin\left(\frac{n\pi}{2}\right) & \cos\left(\frac{n\pi}{2}\right) \\ n! & \sin\left(\frac{n\pi}{2}\right) & \cos\left(\frac{n\pi}{2}\right) \\ a & a^2 & a^3 \end{vmatrix}$

$= 0$ ($\because R_1$ and R_2 are identical)

132 **(a)**

Let, $\Delta = \begin{vmatrix} 1 & \log_x y & \log_x z \\ \log_y x & 1 & \log_y z \\ \log_z x & \log_z y & 1 \end{vmatrix}$

$= 1(1-\log_z y\ \log_y z)$
$\quad -\log_x y\left(\log_y x - \log_y z\ \log_z x\right)$
$\quad +\log_x z\ (\log_y x\ \log_z y - \log_z x$

$= (1-\log_z z) - \log_x y\left(\log_y x - \log_y z\ \log_z x\right)$
$\quad +\log_x z\ (\log_y x \log_z y - \log_z x)$

$= (1-1) - \left(1-\log_x y \log_y x\right) + (\log_x z \log_z x - 1) = 0$ (Since, $\log_x y\ \log_y x = 1$)

$= 0-(1-1)+(1-1) = 0$

133 **(b)**

Given determinant is

$\Delta = \begin{vmatrix} 15! & 16! & 17! \\ 16! & 17! & 18! \\ 17! & 18! & 19! \end{vmatrix}$

Applying $C_2 \to C_2 - C_1, C_3 \to C_3 - C_2$

$\Delta = \begin{vmatrix} 15! & 15\times 15! & 16\times 16! \\ 16! & 16\times 16! & 17\times 17! \\ 17! & 17\times 17! & 18\times 18! \end{vmatrix}$

$= (15!)(16!)(17!)\begin{vmatrix} 1 & 15 & 16\times 16 \\ 1 & 16 & 17\times 17 \\ 1 & 17 & 18\times 18 \end{vmatrix}$

Applying $R_1 \to R_1 - R_2, R_2 \to R_2 - R_3$

$= (15!)(16!)(17!)\begin{vmatrix} 0 & -1 & -33 \\ 0 & -1 & -35 \\ 1 & 17 & 18\times 18 \end{vmatrix}$

$= 2\times(15!)(16!)(17!)$

134 **(b)**

We have,

$\begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \\ 1 & 3 & 6 & 10 \\ 1 & 4 & 10 & 20 \end{vmatrix}$

$$=\begin{vmatrix}1&1&1&1\\0&1&2&3\\0&2&5&9\\0&3&9&19\end{vmatrix}\begin{bmatrix}\text{Applying } R_2\to R_2-R_1,\\ R_3\to R_3-R_1, R_4\to R_4-R_1\end{bmatrix}$$

$$=\begin{vmatrix}1&2&3\\2&5&9\\3&9&19\end{vmatrix}$$

$$=\begin{vmatrix}1&2&3\\0&1&3\\0&3&10\end{vmatrix}\begin{bmatrix}\text{Applying } R_2\to R_2-2R_1,\\ R_3\to R_3-3R_1\end{bmatrix}$$

$=(10-9)=1$

135 **(c)**

The homogenous linear system of equations is consistent *ie*, possesses trivial solution, if $\Delta\equiv$

$$\begin{vmatrix}2&3&5\\1&k&5\\k&-12&-14\end{vmatrix}\neq 0$$

$\Rightarrow 2(-14k+60)-3(-14-5k)+5(-12-k^2)\neq 0$

$\Rightarrow 5k^2+13k-102\neq 0$

$\Rightarrow (5k-17)(k+6)\neq 0$

$\Rightarrow k\neq -6, \dfrac{17}{5}$

136 **(c)**

We have,

$$\begin{vmatrix}b^2+c^2&ab&ac\\ab&c^2+a^2&bc\\ca&c\,b&a^2+b^2\end{vmatrix}$$

$$=\frac{1}{abc}\begin{vmatrix}a(b^2+c^2)&a^2b&a^2c\\ab^2&b(c^2+a^2)&b^2c\\c^2a&c^2b&c(a^2+b^2)\end{vmatrix}$$

[Applying $R_1\to R_1(a), R_2\leftrightarrow R_2(b), R_3\leftrightarrow R_3(c)$]

$$=\frac{1}{abc}abc\begin{vmatrix}b^2+c^2&a^2&a^2\\b^2&c^2+a^2&b^2\\c^2&c^2&a^2+b^2\end{vmatrix}$$

$$=\begin{vmatrix}0&-2\,c^2&-2\,b^2\\b^2&c^2+a^2&b^2\\c^2&c^2&a^2+b^2\end{vmatrix}\text{ Applying } R_1\to R_1-(R_2+R_3)$$

$=4\,a^2b^2c^2$

$\therefore ka^2b^2c^2=4a^2b^2c^2\Rightarrow k=4$

137 **(a)**

We have,

$$\begin{vmatrix}y+z&x&y\\z+x&z&x\\x+y&y&z\end{vmatrix}$$

$$=\begin{vmatrix}2(x+y+z)&x+y+z&x+y+z\\z+x&z&x\\x+y&y&z\end{vmatrix}\text{ App } R_1\to R_1\cdot$$

$$=(x+y+z)\begin{vmatrix}2&1&1\\z+x&z&x\\x+y&y&z\end{vmatrix}$$

$$=(x+y+z)\begin{vmatrix}0&1&1\\0&z&x\\x-z&y&z\end{vmatrix}\text{ Applying } C_1\to C_1-C_2-C_3$$

Hence, the repeating factor is $(z-x)$

138 **(d)**

$$\begin{vmatrix}4+x^2&-6&-2\\-6&9+x^2&3\\-2&3&1+x^2\end{vmatrix}$$

$=(4+x^2)[(1+x^2)(9+x^2)-9]$

$+6[-6(1+x^2)+6]-2[-18+2(9+x^2)]$

$=(4+x^2)(10x^2+x^4)-36x^2-4x^2$

$=40x^2+4x^4+10x^4+x^6-40x^2$

$=x^4(x^2+14)$

Which is not divisible by x^5.

139 **(d)**

Since, for $x=0$, the determinant reduces to the determinant of a skew-symmetric matrix of odd order which is always zero. Hence, $x=0$ is the solution of the given equation.

140 **(c)**

$$\begin{vmatrix}b^2+c^2&a^2&a^2\\b^2&c^2+a^2&b^2\\c^2&c^2&a^2+b^2\end{vmatrix}$$

$$=-2\begin{vmatrix}0&c^2&b^2\\b^2&c^2+a^2&b^2\\c^2&c^2&a^2+b^2\end{vmatrix}[R_1\to R_1-(R_2+R_3)]$$

$$=-2\begin{vmatrix}0&c^2&b^2\\b^2&a^2&0\\c^2&0&a^2\end{vmatrix}\begin{pmatrix}R_2\to R_2-R_1\\R_3\to R_3-R_1\end{pmatrix}$$

$=-2[-c^2(b^2a^2-0)+b^2(0-a^2c^2)]$

$=-2[-2a^2b^2c^2]=4a^2b^2c^2$

141 **(c)**

We have, $\begin{vmatrix}p&b&c\\p+a&q+b&2c\\a&b&r\end{vmatrix}=0$

$\Rightarrow\begin{vmatrix}p&b&c\\p&b&c\\a&b&r\end{vmatrix}+\begin{vmatrix}p&b&c\\a&q&c\\a&b&r\end{vmatrix}=0$

$\Rightarrow 0+\begin{vmatrix}p&b&c\\a&q&c\\a&b&r\end{vmatrix}=0$

$\Rightarrow p(qr-bc)-b(ar-ac)-c(ab-aq)=0$

$\Rightarrow -pqr+pbc+bar+acq=0$

On simplifying, we get

$\dfrac{p}{p-a}+\dfrac{q}{q-b}+\dfrac{r}{r-c}=2$

142 **(d)**

Let $\Delta=\begin{vmatrix}a+b+2c&a&b\\c&2a+b+c&b\\c&a&a+2b+c\end{vmatrix}$

Applying $C_1\to C_1+C_2+C_3$

$$= \begin{vmatrix} 2(a+b+c) & a & b \\ 2(a+b+c) & 2a+b+c & b \\ 2(a+b+c) & a & a+2b+c \end{vmatrix}$$

$$= 2(a+b+c) \begin{vmatrix} 1 & a & b \\ 1 & 2a+b+c & b \\ 1 & a & a+2b+c \end{vmatrix}$$

$$=2(a+b+c) \begin{vmatrix} 0 & -(a+b+c) & 0 \\ 0 & (a+b+c) & -(a+b+c) \\ 1 & a & a+2b+c \end{vmatrix}$$

$\binom{R_1 \to R_1 - R_2}{R_2 \to R_2 - R_3}$

$$= 2(a+b+c)^3 \begin{vmatrix} 0 & -1 & 0 \\ 0 & 1 & -1 \\ 1 & a & a+2b+c \end{vmatrix}$$

$$= 2(a+b+c)^3$$

143 **(c)**

Since, $-1 \le x < 0$

$\therefore [x] = -1$

Also, $0 \le y < 1 \Rightarrow [y] = 0$

and $1 \le z < 2 \Rightarrow [z] = 1$

$\therefore$ Given determinant becomes

$$\begin{vmatrix} 0 & 0 & 1 \\ -1 & 1 & 1 \\ -1 & 0 & 2 \end{vmatrix} = 1 = [z]$$

144 **(b)**

For singular matrix,

$$\begin{bmatrix} -x & x & 2 \\ 2 & x & -x \\ x & -2 & -x \end{bmatrix} = 0$$

Applying $C_2 \to C_2 + C_1, C_3 \to C_3 + C_1$

$$\Rightarrow \begin{vmatrix} -x & 0 & 2-x \\ 2 & 2+x & 2-x \\ x & x-2 & 0 \end{vmatrix} = 0$$

$$\Rightarrow (2-x) \begin{vmatrix} -x & 0 & 1 \\ 2 & 2+x & 1 \\ x & x-2 & 0 \end{vmatrix} = 0$$

Applying $R_2 \to R_2 - R_1$

$$\Rightarrow (2-x) \begin{vmatrix} -x & 0 & 1 \\ 2+x & 2+x & 0 \\ x & x-2 & 0 \end{vmatrix} = 0$$

$$\Rightarrow (2-x)(2+x) \begin{vmatrix} -x & 0 & 1 \\ 1 & 1 & 0 \\ x & x-2 & 0 \end{vmatrix} = 0$$

$$\Rightarrow (2-x)(2+x)(x-2-x) = 0$$

$$\Rightarrow x = 2, -2$$

$\therefore$ Given matrix is non $-$

singular for all x other than 2 and -2.

146 **(c)**

$$\begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} + (-1)^n \begin{vmatrix} a+1 & b+1 & c-1 \\ a-1 & b-1 & c+1 \\ a & -b & c \end{vmatrix}^1$$

$$= \begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} + (-1)^n \begin{vmatrix} a+1 & a-1 & a \\ b+1 & b-1 & -b \\ c-1 & c+1 & c \end{vmatrix}$$

$$== \begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} + (-1)^{n+1} \begin{vmatrix} a+1 & a & a-1 \\ b+1 & -b & b-1 \\ c-1 & c & c+1 \end{vmatrix}$$

$C_2 \leftrightarrow C_3$

$$= (1 + (-1)^{n+2}) \begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix}$$

This is equal to zero only, if $n+2$ is odd *ie*, n is an odd integer.

147 **(d)**

$$\text{Given that, } \begin{vmatrix} \frac{1}{a} & a^2 & bc \\ \frac{1}{b} & b^2 & ca \\ \frac{1}{c} & c^2 & ab \end{vmatrix} = \frac{1}{abc} \begin{vmatrix} 1 & a^3 & abc \\ 1 & b^3 & abc \\ 1 & c^3 & abc \end{vmatrix}$$

$$= \frac{abc}{abc} \begin{vmatrix} 1 & a^3 & 1 \\ 1 & b^3 & 1 \\ 1 & c^3 & 1 \end{vmatrix}$$

$$= 0$$

($\because$ columns C_1 and C_3 are same)

148 **(b)**

$$\text{Given that, } \begin{vmatrix} x & -6 & -1 \\ 2 & -3x & x-3 \\ -3 & 2x & x+2 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x & -6 & -1 \\ 5 & -5x & -5 \\ -3 & 2x & x+2 \end{vmatrix} = 0 \ (R_2 \to R_2 - R_3)$$

$$\Rightarrow 5 \begin{vmatrix} x & -6 & -1 \\ 1 & -x & -1 \\ -3 & 2x & x+2 \end{vmatrix} = 0$$

$$\Rightarrow x(-x^2 - 2x + 2x) - 1(-6x - 12 + 2x) - 3(6 - x) = 0$$

$$\Rightarrow -x^3 + 7x - 6 = 0$$

$$\Rightarrow x^3 - 7x + 6 = 0$$

$$\Rightarrow (x-1)(x-2)(x+3) = 0$$

$$\Rightarrow x = 1, 2, -3$$

$\therefore$ Option (b) is correct.

149 **(a)**

$$\begin{vmatrix} a^2 & b^2 & c^2 \\ (a+1)^2 & (b+1)^2 & (c+1)^2 \\ (a-1)^2 & (b-1)^2 & (c-1)^2 \end{vmatrix}$$

Applying $R_2 \to R_2 - R_3$

$$= \begin{vmatrix} a^2 & b^2 & c^2 \\ 4a & 4b & 4c \\ (a-1)^2 & (b-1)^2 & (c-1)^2 \end{vmatrix}$$

$$= 4\begin{vmatrix} a^2 & b^2 & c^2 \\ a & b & c \\ (a-1)^2 & (b-1)^2 & (c-1)^2 \end{vmatrix}$$

Applying $R_3 \to R_3 - (R_1 - 2R_2)$

$$= 4\begin{vmatrix} a^2 & b^2 & c^2 \\ a & b & c \\ 1 & 1 & 1 \end{vmatrix}$$

151 **(a)**

Given, $A = \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}$

$\Rightarrow |A| = 5 - 6 = -1$

$\therefore |A^{2009} - 5A^{2008}| = |A^{2008}||A - 5I|$

$= (-1)^{2008}\left|\begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix} - \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}\right|$

$= \begin{vmatrix} -4 & 2 \\ 3 & 0 \end{vmatrix} = -6$

152 **(b)**

$$f(1) = \begin{vmatrix} -2 & -16 & -78 \\ -4 & -48 & -496 \\ 1 & 2 & 3 \end{vmatrix} = 2928$$

$$f(3) = \begin{vmatrix} 0 & 0 & 0 \\ -2 & -32 & -392 \\ 1 & 2 & 3 \end{vmatrix} = 0$$

and $f(5) = \begin{vmatrix} 2 & 32 & 294 \\ 0 & 0 & 0 \\ 1 & 2 & 3 \end{vmatrix} = 0$

$\therefore\ f(1).f(3) + f(3).f(5) + f(5).f(1)$

$= f(1).0 + 0 + f(1).0 = 0 = f(3)$ or $f\ (5)$

153 **(d)**

$$\Delta = (x + a + b + c)\begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix} \quad [C_1 \to C_1 + C_2 + C_3]$$

$$= (x + a + b + c)(a + b + c)\begin{vmatrix} 1 & 1 & b+c \\ 1 & 1 & c+b \\ 1 & 1 & a+b \end{vmatrix}$$

$= 0 \quad [C_2 \to C_2 + C_3]$

Hence, x may have any value.

154 **(c)**

It has a non-zero solution, if $\begin{vmatrix} 1 & k & -1 \\ 3 & -k & -1 \\ 1 & -3 & 1 \end{vmatrix} = 0$

$\Rightarrow 1(-k - 3) - k(3 + 1) - 1(-9 + k) = 0$

$\Rightarrow\ -6k + 6 = 0$

$\Rightarrow k = 1$

155 **(a)**

Given, $\begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0$

$$\Rightarrow \begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix} + \begin{vmatrix} x & x^2 & x^3 \\ y & y^2 & y^3 \\ z & z^2 & z^3 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix} + xyz\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} = 0$$

$$\Rightarrow (1 + xyz)\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} = 0$$

$\Rightarrow 1 + xyz = 0$

$\Rightarrow\ xyz = -1$

156 **(a)**

$$\begin{vmatrix} [e] & [\pi] & [\pi^2 - 6] \\ [\pi] & [\pi^2 - 6] & [e] \\ [\pi^2 - 6] & [e] & [\pi] \end{vmatrix}$$

$$= \begin{vmatrix} 2 & 3 & 3 \\ 3 & 3 & 2 \\ 3 & 2 & 3 \end{vmatrix}$$

$= 2(9 - 4) - 3(9 - 6) + 3(6 - 9)$

$= 10 - 9 - 9$

$= -8$

157 **(b)**

We have,

$$\Delta = \begin{vmatrix} a & b & a\,x + b \\ b & c & b\,x + c \\ a\,x + b & b\,x + c & 0 \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} a & b & a\,x + b \\ b & c & b\,x + c \\ 0 & 0 & -(a\,x^2 + 2\,b\,x + c) \end{vmatrix},$$

$\begin{bmatrix} \text{Applying } R_3 \to R_3 - x \\ R_1 - R_2 \end{bmatrix}$

$\Rightarrow \Delta = (b^2 - ac)(ac^2 + 2\,b\,x + c)$

$\therefore \Delta = 0$

$\Rightarrow b^2 = ac$ or, $a\,x^2 + 2\,b\,x + c = 0$

$\Rightarrow a, b, c$ are in G.P. or, x is a root of the equation $ax^2 + 2b\,x + c = 0$

158 **(d)**

All statements are false.

159 **(b)**

Applying $C_3 \to C_3 - C_1$, we get

$$\Delta = \begin{vmatrix} 1 & \alpha & \alpha^2 - 1 \\ \cos(p - d)\,a & \cos pa & 0 \\ \sin(p - d)\,a & \sin pa & 0 \end{vmatrix}$$

$= (\alpha^2 - 1)\{\sin pa \cos(p - d)\,a - \cos pa \sin(p - d)\,a\}$

$= (\alpha^2 - 1)\sin\{-(p - d)a + pa\}$

$\Rightarrow \Delta = (\alpha^2 - 1)\sin da$

Which is independent of p.

160 **(c)**

Given, $\begin{vmatrix} a+x & a-x & a-x \\ a-x & a+x & a-x \\ a-x & a-x & a+x \end{vmatrix} = 0$

Applying $C_1 \to C_1 + C_2 + C_3$ and taking common $(3a - x)$ from C_1, we get

$$(3a - x)\begin{vmatrix} 1 & a-x & a-x \\ 1 & a+x & a-x \\ 1 & a-x & a+x \end{vmatrix} = 0$$

$$\Rightarrow (3a-x)\begin{vmatrix}1 & a-x & a-x\\ 0 & 2x & 0\\ 0 & 0 & 2x\end{vmatrix} = 0\begin{bmatrix}R_2\to R_2-R_1\\ R_3\to R_3-R_1\end{bmatrix}$$

$\Rightarrow (3a-x)(4x^2)=0$

$\Rightarrow x=3a,0$

161 **(a)**

Since, the given equations are consistent.

$$\therefore \begin{vmatrix}2 & 3 & 1\\ 3 & 1 & -2\\ a & 2 & -b\end{vmatrix}=0$$

$\Rightarrow 2(-b+4)-3(-3b+2a)+1(6-a)=0$

$\Rightarrow -2b+8+9b-6a+6-a=0$

$\Rightarrow 7b-7a=-14$

$\Rightarrow a-b=2$

162 **(d)**

Given, $\Delta(x)=$

$$\begin{vmatrix}1 & \cos x & 1-\cos x\\ 1+\sin x & \cos x & 1+\sin x-\cos x\\ \sin x & \sin x & 1\end{vmatrix}$$

Applying $C_3 \to C_3 + C_2 - C_1$

$$=\begin{vmatrix}1 & \cos x & 0\\ 1+\sin x & \cos x & 0\\ \sin x & \sin x & 1\end{vmatrix}$$

$=\cos x-\cos x\,(1+\sin x)$

$=-\cos x\,\sin x$

$=-\frac{1}{2}\sin 2x$

$$\therefore \int_0^{\pi/2}\Delta x\,dx=-\frac{1}{2}\int_0^{\pi/2}\sin 2x\,dx$$

$$=-\frac{1}{2}\left[-\frac{\cos 2x}{2}\right]_0^{\pi/2}=-\frac{1}{2}$$

163 **(c)**

For the non-trivial solution, we must have

$$\begin{vmatrix}1 & a & a\\ b & 1 & b\\ c & c & 1\end{vmatrix}=0$$

$$\Rightarrow \begin{vmatrix}1-a & 0 & a\\ b-1 & 1-b & b\\ 0 & c-1 & 1\end{vmatrix}=0$$

$\begin{bmatrix}\text{Applying } C_1\to C_1-C_2;\\ C_2\to C_2-C_3\end{bmatrix}$

$\Rightarrow (1-a)[(1-b)-b(c-1)]+a(b-1)(c-1)=0$

$$\Rightarrow \frac{1}{c-1}+\frac{b}{b-1}+\frac{a}{a-1}=0$$

$$\Rightarrow \left(\frac{1}{c-1}+1\right)+\frac{b}{b-1}+\frac{a}{a-1}=1$$

$$\Rightarrow \frac{c}{c-1}+\frac{b}{b-1}+\frac{a}{a-1}=1$$

$$\Rightarrow \frac{a}{1-a}+\frac{b}{1-b}+\frac{c}{1-c}=-1$$

164 **(d)**

Given system equations are

$3x-2y+z=0$

$\lambda x-14y+15z=0$ and $x+2y-3z=0$

The system of equations has infinitely many (non-trivial solutions, if $\Delta=0$.

$$\Rightarrow \Delta=\begin{vmatrix}3 & -2 & 1\\ \lambda & -14 & 15\\ 1 & 2 & -3\end{vmatrix}=0$$

$\Rightarrow 3(42-30)-\lambda(6-2)+1(-30+14)=0$

$\Rightarrow 36-4\lambda-16=0$

$\Rightarrow \lambda=5$

166 **(c)**

$$\text{Since, }\begin{vmatrix}\sin x & \cos x & \cos x\\ \cos x & \sin x & \cos x\\ \cos x & \cos x & \sin x\end{vmatrix}=0$$

$\Rightarrow \sin x(\sin^2 x-\cos^2 x)$
$\quad -\cos x(\cos x\sin x-\cos^2 x)$
$+\cos x(\cos^2 x-\sin x\cos x)=0$

$\Rightarrow \sin x(\sin^2 x-\cos^2 x)$
$\quad -2\cos^2 x(\sin x-\cos x)=0$

$\Rightarrow (\sin x-\cos x)[\sin x(\sin x$
$\quad +\cos x)-2\cos^2 x]=0$

$\Rightarrow (\sin x-\cos x)[(\sin^2 x-\cos^2 x)$
$\quad +(\sin x\cos x-\cos^2 x)]=0$

$\Rightarrow (\sin x-\cos x)^2\,[\sin x+\cos x+\cos x]=0$

$\Rightarrow (\sin x-\cos x)^2\,(\sin x+2\cos x)=0$

$\Rightarrow$ Either $(\sin x-\cos x)^2=0$

or $\sin x+2\cos x)=0$

$\Rightarrow$Either $\tan x=1$ or $\tan x=-2$

$\Rightarrow$ Either $x=\frac{\pi}{4}$ or $\tan x=-2$

As $x\in\left[-\frac{\pi}{4},\frac{\pi}{4}\right]$, $\tan x\in[-1,1]$

Hence, real solution is only $x=\frac{\pi}{4}$

167 **(a)**

Applying $R_1\to R_1+R_3-2R_2$, we get

$$\Delta=\begin{vmatrix}0\;0\;0 & x+z-zy\\ 4\;5\;6 & y\\ 5\;6\;7 & z\\ x\;y\;z & 0\end{vmatrix}$$

$$=-(x+z-2y)\begin{vmatrix}4 & 5 & 6\\ 5 & 6 & 7\\ x & y & z\end{vmatrix}\ \text{[Expanding along } R_1]$$

$$=-(x+z-2y)\begin{vmatrix}0 & -1 & 6\\ 0 & -1 & 7\\ x-2y+z & y-z & z\end{vmatrix}$$

$\begin{bmatrix}\text{Applying } C_1\to C_1+C_3\\ -2C_2 \text{ and } C_2\to C_2-C_3\end{bmatrix}$

$$=-(x+z-2y)^2\begin{vmatrix}-1 & 6\\ -1 & 7\end{vmatrix}=(x-2y+z)^2$$

169 **(c)**

We have, $a=1+2+4+8+\ldots$ upto n terms

$= 1\left(\frac{2^n - 1}{2 - 1}\right) = 2^n - 1$

$b = 1 + 3 + 9 + \ldots$ upto n terms$=\frac{3^n-1}{2}$

and $c = 1 + 5 + 25 + \ldots$ upto n terms$=\frac{5^n-1}{4}$

$\therefore \begin{vmatrix} a & 2b & 4c \\ 2 & 2 & 2 \\ 2^n & 3^n & 5^n \end{vmatrix} = 2\begin{vmatrix} 2^n - 1 & 3^n - 1 & 5^n - 1 \\ 1 & 1 & 1 \\ 2^n & 3^n & 5^n \end{vmatrix}$

$= 2\begin{vmatrix} 2^n & 3^n & 5^n \\ 1 & 1 & 1 \\ 2^n & 3^n & 5^n \end{vmatrix}$ $[R_1 \to R_1 + R_2]$

$= 2 \times 0 = 0$ [$\therefore$ two rows are identical]

170 **(d)**

Let $\Delta = \begin{vmatrix} c & 1 & 0 \\ 1 & c & 1 \\ 6 & 1 & c \end{vmatrix} = c(c^2 - 1) - 1(c - 6)$

$= 8\cos^3\theta - 4\cos\theta + 6$

171 **(b)**

We have,

$\Delta = \begin{vmatrix} 1^2 & 2^2 & 3^2 \\ 2^2 & 3^2 & 4^2 \\ 3^2 & 4^2 & 5^2 \end{vmatrix}$

$\Rightarrow \Delta = \begin{vmatrix} 1 & 3 & 5 \\ 4 & 5 & 7 \\ 9 & 7 & 9 \end{vmatrix}$ Applying $C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_2$

$\Rightarrow \Delta = \begin{vmatrix} 1 & 3 & 2 \\ 4 & 5 & 2 \\ 9 & 7 & 2 \end{vmatrix}$ Applying $C_3 \to C_3 - C_2$

$\Rightarrow \Delta = 2\begin{vmatrix} 1 & 3 & 1 \\ 4 & 5 & 1 \\ 9 & 7 & 1 \end{vmatrix}$

$\Rightarrow \Delta = 2\begin{vmatrix} 1 & 3 & 1 \\ 3 & 2 & 0 \\ 8 & 4 & 0 \end{vmatrix}$ Applying $R_2 \to R_2 - R_1, R_3 \to R_3 - R_1$

$\Rightarrow \Delta = 2 \times -4 = -8$

172 **(a)**

We have,

$\begin{vmatrix} x & a & b \\ a & x & b \\ a & b & x \end{vmatrix}$

$= \begin{vmatrix} x & a & b \\ a - x & x - a & 0 \\ a - x & b - a & x - b \end{vmatrix}$

$\begin{bmatrix} \text{Applying } R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{bmatrix}$

$= (x - a)\begin{vmatrix} x & a & b \\ -1 & 1 & 0 \\ a - x & b - a & x - b \end{vmatrix}$

$= (x - a)\begin{vmatrix} x + a + b & a & b \\ 0 & 1 & 0 \\ 0 & b - a & x - b \end{vmatrix}$

$\begin{bmatrix} \text{Applying} \\ C_1 \to C_1 + C_2 + C_3 \end{bmatrix}$

$= (x - a)(x + a + b)(x - b)$ [Expanding along C_1]

173 **(c)**

We have,

$\Delta = \begin{vmatrix} [x] + 1 & [y] & [z] \\ [x] & [y] + 1 & [z] \\ [x] & [y] & [z] + 1 \end{vmatrix}$

$\Rightarrow \Delta = \begin{vmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ [x] & [y] & [z] + 1 \end{vmatrix}$

$\begin{bmatrix} \text{Applying } R_1 \to R_1 - R_3 \\ R_2 \to R_2 - R_3 \end{bmatrix}$

$\Rightarrow \Delta = [z] + 1 + [y] + [x] = [x] + [y] + [z] + 1$

Since maximum values of $[x]$, $[y]$ and $[z]$ are 1, 0 and 2 respectively

$\therefore$ Maximum value of $\Delta = 2 + 1 + 0 + 1 = 4$

174 **(c)**

We have,

$\begin{vmatrix} a & 2b & 2c \\ 3 & b & c \\ 4 & a & b \end{vmatrix} = 0$

$\Rightarrow \begin{vmatrix} a - 6 & 0 & 0 \\ 3 & b & c \\ 4 & a & b \end{vmatrix} = 0$ Applying $R_1 \to R_1 - 2R_2$

$\Rightarrow (a - 6)(b^2 - ac) = 0 \Rightarrow b^2 = ac \Rightarrow b^3 = abc$

176 **(d)**

We have, $\Delta \equiv \begin{vmatrix} a & b & 0 \\ 0 & a & b \\ b & 0 & a \end{vmatrix} = 0$

$\Rightarrow \Delta \equiv a(a^2 - 0) - b(0 - b^2) = a^3 + b^3$

$\Rightarrow a^3 + b^3 = 0 \Rightarrow \left(\frac{a}{b}\right)^3 = -1$

$\therefore \left(\frac{a}{b}\right)$ is one of the cube roots of -1.

177 **(b)**

We have,

$\begin{vmatrix} b + c & c + a & a + b \\ a + b & b + c & c + a \\ c + a & a + b & b + c \end{vmatrix} = k\begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix}$

Applying $C_1 \leftarrow C_1 + (C_2 + C_3)$ on LHS, we have

$\Rightarrow \begin{vmatrix} 2(a + b + c) & c + a & a + b \\ 2(a + b + c) & b + c & c + a \\ 2(a + b + c) & a + b & b + c \end{vmatrix} = k\begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix}$

$\Rightarrow 2\begin{vmatrix} a + b + c & c + a & a + b \\ a + b + c & b + c & c + a \\ a + b + c & a + c & b + c \end{vmatrix} = k\begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix}$

Applying $C_2 \to C_2 - C_1, C_3 \to C_3 - C_1$ on LHS, we have

$\Rightarrow 2\begin{vmatrix} a + b + c & -b & -c \\ a + b + c & -a & -b \\ a + b + c & -c & -a \end{vmatrix} = k\begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix}$

Applying $C_1 \to C_1 + C_2 + C_3$ on LHS, we have

$\Rightarrow \begin{vmatrix} a & -b & -c \\ c & -a & -b \\ b & -c & -a \end{vmatrix} = k\begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix}$

$$\Rightarrow 2\begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix} = k\begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix}$$

$\therefore k = 2$

178 **(b)**

Let $\Delta = \begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix} = abc - (a+b+c) + 2$

$\because \Delta > 0 \Rightarrow abc + 2 > a + b + c$

$\Rightarrow abc + 2 > 3(abc)^{1/3}$

$\left[\because \text{AM} > \text{GM} \Rightarrow \frac{a+b+c}{0} > (abc)^{1/3}\right]$

$\Rightarrow x^3 + 2 > 3x$, where $x = (abc)^{1/3}$

$\Rightarrow x^3 - 3x + 2 > 0 \Rightarrow (x-1)^2(x+2) > 0$

$\Rightarrow x + 2 > 0 \Rightarrow x > -2 \Rightarrow (abc)^{1/3} > -2$

$\Rightarrow abc > -8$

179 **(a)**

Applying $R_3 \to R_3 - R_1(\cos\beta) + R_2(\sin\beta)$

$$\begin{vmatrix} \cos\alpha & -\sin\alpha & 1 \\ \sin\alpha & \cos\alpha & 1 \\ 0 & 0 & 1 + \sin\beta - \cos\beta \end{vmatrix}$$

$= (1 + \sin\beta - \cos\beta)(\cos^2\alpha + \sin^2\alpha)$

$= 1 + \sin\beta - \cos\beta$, which is independent of α

180 **(d)**

Given, $A = B^{-1}AB$

$\Rightarrow BA = AB$

$\therefore \det(B^{-1}AB) = \det(B^{-1}BA) = \det(A)$

181 **(d)**

Given, matrix is singular.

Therefore, $\begin{vmatrix} 0 & 1 & -2 \\ -1 & 0 & 3 \\ \lambda & -3 & 0 \end{vmatrix} = 0$

$\Rightarrow +1(0-6) + \lambda(3) = 0$

$\Rightarrow -6 + 3\lambda = 0$

$\Rightarrow \lambda = 2$

182 **(a)**

We have,

$$|A| = \begin{vmatrix} 4 & 5 & 6 & x \\ 5 & 6 & 7 & y \\ 6 & 7 & 8 & z \\ x & y & z & 0 \end{vmatrix}$$

$$\Rightarrow |A| = \begin{vmatrix} 4 & 5 & 6 & x \\ 10 & 12 & 14 & 2y \\ 6 & 7 & 8 & z \\ x & y & z & 0 \end{vmatrix} \text{[Applying } R_2 \to 2R_2]$$

$$\Rightarrow |A| = \begin{vmatrix} 4 & 5 & 6 & x \\ 0 & 0 & 0 & 0 \\ 6 & 7 & 8 & z \\ x & y & z & 0 \end{vmatrix} \text{[Applying } R_2 \to R_2 - (R_1 + R_3)]$$

$\Rightarrow |A| = 0 \quad [\because 2y = x + z]$

183 **(c)**

Putting $r = 1, 2, 3, \dots, n$ and using the formula

$\sum 1 = n$ and $\sum r = \frac{(n+1)n}{2}$

$\sum(2r-1) = 1 + 3 + 5 + \ldots = n^2$

$$\therefore \sum_{r=1}^{n} \Delta_r = \begin{vmatrix} n & n & n \\ n(n+1) & n^2+n+1 & n^2+n \\ n^2 & n^2 & n^2+n+1 \end{vmatrix} = 56$$

Applying $C_1 \to C_1 - C_3, C_2 \to C_2 - C_3$

$$\begin{vmatrix} 0 & 0 & n \\ 0 & 1 & n^2+n \\ -n-1 & -n-1 & n^2+n+1 \end{vmatrix}$$

$\Rightarrow n(n+1) = 56$

$\Rightarrow n^2 + n - 56 = 0$

$\Rightarrow (n+8)(n-7) = 0$

$\Rightarrow n = 7 \quad (n \neq -8)$

184 **(a)**

$$\begin{vmatrix} 1 & a & a^2 - bc \\ 1 & b & b^2 - ac \\ 1 & c & c^2 - ab \end{vmatrix} =$$

$$\begin{vmatrix} 0 & a-b & (a-b)(a+b+c) \\ 0 & b-c & (b-c)(a+b+c) \\ 1 & c & c^2 - ab \end{vmatrix} \begin{pmatrix} R_1 \to R_1 - R_2 \\ R_2 \to R_2 - R_3 \end{pmatrix}$$

$= (a-b)(b-c)\begin{vmatrix} 0 & 1 & a+b+c \\ 0 & 1 & a+b+c \\ 1 & c & c^2 - ab \end{vmatrix} = 0$ ($\because$ rows R_1 and R_2 are identical)

185 **(c)**

$\because \det(M_r) = r^2 - (r-1)^2 = 2r - 1$

$\therefore \det(M_1) + \det(M_2) + \ldots + \det(M_{2008})$

$= 1 + 3 + 5 + \ldots + 4015$

$= \frac{2008}{2}[2 + (2008-1)2]$

$= 2008(2008) = (2008)^2$

186 **(b)**

$$\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix}$$

Applying $C_1 \to C_1 + C_2 + C_3$

$$= \begin{vmatrix} 1+\omega+\omega^2 & \omega & \omega^2 \\ 1+\omega+\omega^2 & \omega^2 & 1 \\ 1+\omega+\omega^2 & 1 & \omega \end{vmatrix} (\because 1 + \omega + \omega^2 = 0)$$

$$= \begin{vmatrix} 0 & \omega & \omega^2 \\ 0 & \omega^2 & 1 \\ 0 & 1 & \omega \end{vmatrix}$$

$= 0$

187 **(a)**

Given, $\Delta = \begin{vmatrix} 1 & \omega^n & \omega^{2n} \\ \omega^n & \omega^{2n} & 1 \\ \omega^{2n} & 1 & \omega^n \end{vmatrix}$

$= 1(\omega^{3n} - 1) - \omega^n(\omega^{2n} - \omega^{2n}) + \omega^{2n}(\omega^n - \omega^{4n})$

$= 1(1-1) - 0 + \omega^{2n}(\omega^n - \omega^n) \quad [\because \omega^3 - 1]$

$= 0$

188 **(a)**

Given, $\Delta=\begin{vmatrix}1 & 1 & 1\\ a & b & c\\ a^3 & b^3 & c^3\end{vmatrix}=$

$\begin{vmatrix}1 & 0 & 0\\ a & a-b & a-c\\ a^3 & a^3-b^3 & a^3-c^3\end{vmatrix}$

$[C_2 \to C_1 - C_2, C_3 \to C_1 - C_3]$

$=(a-b)(a-c)\begin{vmatrix}1 & 0 & 0\\ a & 1 & 1\\ a^3 & a^2+ab+b^2 & a^2+ac+c^2\end{vmatrix}$

$=(a-b)(a-c)(c^2+ac-ab-b^2)$

$=(a-b)(b-c)(c-a)(a+b+c)$

189 **(c)**

We have,

$\Delta=\begin{vmatrix}b+c & a & a\\ b & c+a & b\\ c & c & a+b\end{vmatrix}$

$\Rightarrow \Delta=\begin{vmatrix}2(b+c) & 2(c+a) & 2(a+b)\\ b & c+a & b\\ c & c & a+b\end{vmatrix}$

Applying $R_1 \to R_1 + 2R_2 + R_3$

$\Rightarrow \Delta=2\begin{vmatrix}b+c & c+a & a+b\\ b & c+a & b\\ c & c & a+b\end{vmatrix}$

$\Rightarrow \Delta=2\begin{vmatrix}b+c & c+a & a+b\\ -c & 0 & -a\\ -b & -a & 0\end{vmatrix}$

Applying $R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$

$\Rightarrow \Delta=2\{(b+c)(0-a^2)-(c+a)(0-ab)+(a+b)(ac-0)\}$

$\Rightarrow \Delta=2\{-a^2(b+c)+ab(c+a)+ac(a+b)\}$

$\Rightarrow \Delta=2(-a^2b-a^2c+abc+a^2b+a^2c+abc)$

$\Rightarrow \Delta=4\,abc$

190 **(d)**

$\begin{vmatrix}\log_5 729 & \log_3 5\\ \log_5 27 & \log_9 25\end{vmatrix}=\begin{vmatrix}\log_3 3^6 & \log_3 5\\ \log_5 3^3 & \log_{3^2} 5^2\end{vmatrix}$

$=\begin{vmatrix}6\log_5 3 & \log_3 5\\ 3\log_5 3 & \frac{2}{2}\log_3 5\end{vmatrix}$

$=6\log_5 3\log_3 5 - 3\log_5 3\log_3 5$

$=6-3=3$

And $\begin{vmatrix}\log_3 5 & \log_{27} 5\\ \log_5 9 & \log_5 9\end{vmatrix}=\begin{vmatrix}\log_3 5 & \log_{3^3} 5\\ \log_5 3^2 & \log_5 3^2\end{vmatrix}$

$=\begin{vmatrix}\log_3 5 & \frac{1}{3}\log_3 5\\ 2\log_5 3 & 2\log_5 3\end{vmatrix}$

$=2\log_5 3\log_3 5-\frac{2}{3}\log_5 3\log_3 5$

$=2-\frac{2}{3}=\frac{4}{3}$

Now, $\begin{vmatrix}\log_5 729 & \log_3 5\\ \log_5 27 & \log_9 25\end{vmatrix}\begin{vmatrix}\log_3 5 & \log_{27} 5\\ \log_5 9 & \log_5 9\end{vmatrix}=$

$3.\frac{4}{3}=4$

Take option(d),

$\log_3 5.\log_5 81=\log_3 81=\log_3 3^4=4$

191 **(c)**

Given, $\begin{vmatrix}a-x & c & b\\ c & b-x & a\\ b & a & c-x\end{vmatrix}=0$

Applying $R_1 \to R_1+R_2+R_3$

$\Rightarrow (a+b+c-x)\begin{vmatrix}1 & 1 & 1\\ c & b-x & a\\ b & a & c-x\end{vmatrix}=0$

$\Rightarrow (a+b+c-x)\begin{vmatrix}1 & 0 & 0\\ c & b-x-c & a-c\\ b & a-b & c-x-b\end{vmatrix}=0$

$\Rightarrow (a+b+c-x)[1(b-x-c)(c-x-b)-(a-c)(a-b)]=0$

$\Rightarrow (a+b+c-x)[bc-xb-b^2-xc+x^2+bx-c^2+cx+bc-(a^2-ab-ac+bc)]=0$

$\Rightarrow (a+b+c-x)[x^2-a^2-b^2-c^2+ab+bc+ca]=0$

$\Rightarrow x=a+b+c$ or $x^2=a^2+b^2+c^2+ab+bc+ca$

$\Rightarrow x=0$ or $x^2=a^2+b^2+c^2+\frac{1}{2}(a^2+b^2+c^2)$

$\Rightarrow x=0$ or $x=\pm\sqrt{\frac{3}{2}(a^2+b^2+c^2)}$

192 **(d)**

We have,

$\Delta=abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)=abc\times 0$

193 **(a)**

Applying $R_1 \to R_1+R_3$, we get

$\begin{vmatrix}1-i & \omega^2+\omega & \omega^2-1\\ 1-i & -1 & \omega^2-1\\ -i & -1+\omega-i & -1\end{vmatrix}=0$

$[\because \omega^2+\omega=-1$, so R_1 and R_2 become identical$]$

194 **(a)**

$\sum_{n=1}^{N} U_n=\begin{vmatrix}\sum n & 1 & 5\\ \sum n^2 & 2N+1 & 2N+1\\ \sum n^3 & 3N^2 & 3N\end{vmatrix}$

$=\begin{vmatrix}\frac{N(N+1)}{2} & 1 & 5\\ \frac{N(N+1)(2N+1)}{6} & 2N+1 & 2N+1\\ \left(\frac{N(N+1)}{2}\right)^2 & 3N^2 & 3N\end{vmatrix}$

$$= \frac{N(N+1)}{12}\begin{vmatrix} 6 & 1 & 5 \\ 4N+2 & 2N+1 & 2N+1 \\ 3N(N+1) & 3N^2 & 3N \end{vmatrix}$$

Applying $C_3 \to C_3 + C_2$

$$= \frac{N(N+1)}{12}\begin{vmatrix} 6 & 1 & 6 \\ 4N+2 & 2N+1 & 4N+2 \\ 3N(N+1) & 3N^2 & 3N(N+1) \end{vmatrix}$$

$= 0$ ($\because$ two columns are identical)

195 **(c)**

$$\begin{bmatrix} 215 & 342 & 511 \\ 6 & 7 & 8 \\ 36 & 49 & 54 \end{bmatrix}$$

$= 215(378 - 392) - 342(324 - 288) + 511(294 - 252)$

$= -3010 - 12312 + 21462 = 6140$

Which is exactly divisible by 20

196 **(a)**

$\det(A^{-1}\text{adj } A) = \det(A^{-1})\det(\text{adj } A)$

$= (\det A)^{-1}(\det A)^{3-1} = \det A$

197 **(d)**

$$A = \begin{vmatrix} 1 & \sin\theta & 1 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{vmatrix}$$

$= 1(1+\sin^2\theta) - \sin\theta(-\sin\theta + \sin\theta) + 1(\sin^2\theta + 1)$

$= 2(1+\sin^2\theta)$

Since, the maximum and minimum value of $\sin^2\theta$ is 1 and 0.

$\therefore |A| \in [2,4]$

198 **(d)**

Since, the first column consists of sum of two terms, second column consists of sum of three terms and third column consists of sum four terms.

$\therefore n = 2 \times 3 \times 4 = 24$

199 **(c)**

Given ,$a_1, a_2, a_3, \dots \in$GP

$\Rightarrow \log a_1, \log a_2, \dots \in$ AP

$\Rightarrow \log a_n, \log a_{n+1}, \log a_{n+2}, \dots \in$AP

$\Rightarrow \log a_{n+1} = \frac{\log a_n + \log a_{n+2}}{2}$ ...(i)

Similarly, $\log a_{n+4} = \frac{\log a_{n+3} + \log a_{n+5}}{2}$ ...(ii)

and $\log a_{n+7} = \frac{\log a_{n+6} + \log a_{n+8}}{2}$ ...(ii)

Given, $\Delta = \begin{vmatrix} \log a_n & \log a_{n+1} & \log a_{n+2} \\ \log a_{n+3} & \log a_{n+4} & \log a_{n+5} \\ \log a_{n+6} & \log a_{n+7} & \log a_{n+8} \end{vmatrix}$

Applying $C_2 \to C_2 - \frac{C_1 + C_3}{2}$

$$\Delta = \begin{vmatrix} \log a_n & 0 & \log a_{n+2} \\ \log a_{n+3} & 0 & \log a_{n+5} \\ \log a_{n+6} & 0 & \log a_{n+8} \end{vmatrix} = 0$$

200 **(a)**

$$\begin{vmatrix} 1 & \omega & -\omega^2/2 \\ 1 & 1 & 1 \\ 1 & -1 & 0 \end{vmatrix} = -\frac{1}{2}\begin{vmatrix} 1 & \omega & \omega^2 \\ 1 & 1 & -2 \\ 1 & -1 & 0 \end{vmatrix}$$

$= -\frac{1}{2}\begin{vmatrix} 1+\omega+\omega^2 & \omega & \omega^2 \\ 0 & 1 & -2 \\ 0 & -1 & 0 \end{vmatrix}$ $(C_1 \to C_1 + C_2 + C_3)$

$= -\frac{1}{2}\begin{vmatrix} 0 & \omega & \omega^2 \\ 0 & 1 & -2 \\ 0 & -1 & 0 \end{vmatrix}$ $(\because 1+\omega+\omega^2 = 0)$

$= 0$

201 **(a)**

$$\begin{vmatrix} \log e & \log e^2 & \log e^3 \\ \log e^2 & \log e^3 & \log e^4 \\ \log e^3 & \log e^4 & \log e^5 \end{vmatrix}$$

$$= \begin{vmatrix} \log e & 2\log e & 3\log e \\ 2\log e & 3\log e & 4\log e \\ 3\log e & 4\log e & 5\log e \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & 1 \\ 3 & 1 & 1 \end{vmatrix}$$

(Using $C_2 \to C_2 - C_1, C_3 \to C_3 - C_2$)

$= 0[\therefore$ two columns are identical$]$

202 **(b)**

$$\begin{vmatrix} \sqrt{13}+\sqrt{3} & 2\sqrt{5} & \sqrt{5} \\ \sqrt{15}+\sqrt{26} & 5 & \sqrt{10} \\ 3+\sqrt{65} & \sqrt{15} & 5 \end{vmatrix}$$

$$= \begin{vmatrix} \sqrt{13} & 2\sqrt{5} & \sqrt{5} \\ \sqrt{16} & 5 & \sqrt{10} \\ \sqrt{65} & \sqrt{15} & 5 \end{vmatrix} + \begin{vmatrix} \sqrt{3} & 2\sqrt{5} & \sqrt{5} \\ \sqrt{15} & 5 & \sqrt{10} \\ 3 & \sqrt{15} & 5 \end{vmatrix}$$

$$= \sqrt{13}.\sqrt{5}.\sqrt{5}\begin{vmatrix} 1 & 2 & 3 \\ \sqrt{2} & \sqrt{5} & \sqrt{2} \\ \sqrt{5} & \sqrt{3} & \sqrt{5} \end{vmatrix}$$

$$+\sqrt{3}.\sqrt{5}.\sqrt{5}\begin{vmatrix} 1 & 2 & 3 \\ \sqrt{5} & \sqrt{5} & \sqrt{2} \\ \sqrt{3} & \sqrt{3} & \sqrt{5} \end{vmatrix}$$

$$= 0 + 5\sqrt{3}\begin{vmatrix} -1 & 2 & 1 \\ 0 & \sqrt{5} & \sqrt{2} \\ 0 & \sqrt{3} & \sqrt{5} \end{vmatrix} = 5\sqrt{3}(\sqrt{6} - 5)$$

204 **(d)**

We can write $\Delta = \Delta_1 + y_1\Delta_2$, where

$$\Delta_1 = \begin{vmatrix} 1 & 1+x_1y_2 & 1+x_1y_3 \\ 1 & 1+x_2y_2 & 1+x_2y_3 \\ 1 & 1+x_3y_2 & 1+x_3y_3 \end{vmatrix}$$

and $\Delta_2 = \begin{vmatrix} x_1 & 1+x_1y_2 & 1+x_1y_3 \\ x_2 & 1+x_2y_2 & 1+x_2y_3 \\ x_3 & 1+x_3y_2 & 1+x_3y_3 \end{vmatrix}$

In Δ_1, use $C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_1$ so that,

$\Delta_1=\begin{vmatrix}1 & x_1y_2 & x_1y_3\\ 1 & x_2y_2 & x_2y_3\\ 1 & x_3y_2 & x_3y_3\end{vmatrix}=0$ ($\because C_2$ and C_3 are proportional)

In Δ_2, $C_2 \to C_2 - y_2C_1$ and $C_3 \to C_3 - y_3C_1$ to get

$\Delta_2=\begin{vmatrix}x_1 & 1 & 1\\ x_2 & 1 & 1\\ x_3 & 1 & 1\end{vmatrix}=0$ ($\because C_2$ and C_3 are identical)

$\therefore\ \Delta=0$

206 **(c)**

Let $\Delta=\begin{vmatrix}b^2-ab & b-c & bc-ac\\ ab-a^2 & a-b & b^2-ab\\ bc-ac & c-a & ab-a^2\end{vmatrix}$

$=(b-a)(b-a)\begin{vmatrix}b & b-c & c\\ a & a-b & b\\ c & c-a & a\end{vmatrix}$

$=(a-b)^2\begin{vmatrix}b & b & c\\ a & a & b\\ c & c & a\end{vmatrix}$ $(C_2 \to C_2 + C_3)$

$=0$ ($\because$ two columns are same)

208 **(d)**

$\begin{vmatrix}x+1 & \omega & \omega^2\\ \omega & x+\omega^2 & 1\\ \omega^2 & 1 & x+\omega\end{vmatrix}$

$=\begin{vmatrix}x+1+\omega+\omega^2 & \omega & \omega^2\\ x+1+\omega+\omega^2 & x+\omega^2 & 1\\ x+1+\omega+\omega^2 & 1 & x+\omega\end{vmatrix}$ $C_1 \to C_1 + C_2 + C_3)$

$=x\begin{vmatrix}1 & \omega & \omega^2\\ 1 & x+\omega^2 & 1\\ 1 & 1 & x+\omega\end{vmatrix}$ ($\because 1+\omega+\omega^2=0$

$=x[1\{(x+\omega^2)(x+\omega)-1\}+\omega\{1-(x+\omega)\}+\omega^2\{1-(x+\omega^2)\}]$

$=x[(x^2+\omega x+\omega^2x+\omega^3-1+\omega-\omega x-\omega^2+\omega^2-\omega^2x-\omega^4]$

$=x^3$ ($\because \omega^3=1$)

210 **(b)**

Given , $f(x)=\begin{vmatrix}x & 1+\sin x & \cos x\\ 1 & \log(1+x) & 2\\ x^2 & 1+x^2 & 0\end{vmatrix}$

$=x\{-2(1+x^2)\}-(1+\sin x)(-2x^2)+\cos x\{1+x^2-x^2\log(1+x)\}$

$=-2x-2x^3+2x^2+2x^2\sin x+\cos x\{1+x^2-x^2\log(1+x)\}$

$\therefore$ Coefficient of x in $f(x)=-2$.

211 **(c)**

Clearly, the degree of the given determinant is 3. So, there cannot be more that 3 linear factors. Thus, the other factor is a numerical constant. Let it be λ. Then,

$\begin{vmatrix}-2a & a+b & a+c\\ b+a & -2b & b+c\\ c+a & c+b & -2c\end{vmatrix}=\lambda(a+b)(b+c)(c+a)$

Putting $a=0, b=1$ and $c=1$ on both sides, we get

$\begin{vmatrix}0 & 1 & 1\\ 1 & -2 & 2\\ 1 & 2 & -2\end{vmatrix}=\lambda\times1\times2\times1 \Rightarrow 2\lambda \Rightarrow \lambda=4$

212 **(b)**

We have,

$\begin{vmatrix}1 & \omega^2 & \omega^5\\ \omega^3 & 1 & \omega^4\\ \omega^5 & \omega^4 & 1\end{vmatrix}$

$=\begin{vmatrix}1 & 1 & \omega^2\\ 1 & 1 & \omega\\ \omega^2 & \omega & 1\end{vmatrix}$

$=2-(\omega^2-\omega)=2-(-1)=3$

213 **(b)**

Applying $C_1 \to C_1+C_2+C_3$ and taking common $(a+b+c)$ from C_1, we get

$(a+b+c)\begin{vmatrix}1 & b & c\\ 1 & c & a\\ 1 & a & b\end{vmatrix}$

Applying $R_2 \to R_2-R_1$ and $R_3 \to R_3-R_1$, we get

$(a+b+c)\begin{vmatrix}1 & b & c\\ 0 & c-b & a-c\\ 0 & a-b & b-c\end{vmatrix}$

$=(a+b+c)\{-(c-b)^2-(a-b)(a-c)\}$

$=-(a+b+c)\{a^2+b^2+c^2-ab-bc-ca\}$

$=-\frac{1}{2}(a+b+c)\{2a^2+2b^2+2c^2-2ab-2bc-2ac\}$

$=-\frac{1}{2}(a+b+c)\{a-b)^2+(b-c)^2+(c-a)^2\}$

Which is always negative.

214 **(c)**

In a ΔABC, we have

$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$

$\Rightarrow b\sin A=a\sin B\ c\sin A=a\sin C$

$\therefore \begin{vmatrix}a^2 & b\sin A & c\sin A\\ b\sin A & 1 & \cos A\\ c\sin A & \cos A & 1\end{vmatrix}$

$=\begin{vmatrix}a^2 & a\sin B & a\sin C\\ a\sin B & 1 & \cos A\\ a\sin C & \cos A & 1\end{vmatrix}$

$=a^2\begin{vmatrix}1 & \sin B & \sin C\\ \sin B & 1 & \cos A\\ \sin C & \cos A & 1\end{vmatrix}$ Taking a common fr R_1 and C_1 both

$=a^2\{(1-\cos^2 A)-\sin B(\sin B-\cos A\sin C)+\sin C(\sin B\cos A-\sin C)\}$

$=a^2\{\sin^2 A-\sin^2 B+2\sin B\sin C\cos A-\sin^2 C\}$

$= a^2\{\sin(A+B)\sin(A-B) - \sin^2 C$
$+ 2\cos A \sin B \sin C\}$
$= a^2[\sin C\{\sin(A-B) - \sin C\}$
$+ 2\cos A \sin B \sin C]$
$= a^2[\sin C\{\sin(A-B) - \sin(A+B)\}$
$+ 2\cos A \sin B \sin C]$
$= a^2[\sin C \times -2\cos A \sin B + 2\cos A \sin B \sin C]$
$= 0$

215 **(b)**

$$\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix} = \begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix} + \begin{vmatrix} a & a^2 & a^3 \\ b & b^2 & b^3 \\ c & c^2 & c^3 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix} + abc \begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix} = 0$$

$$\Rightarrow (1+abc) \begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix} = 0$$

$$\left[\because \begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix} \neq 0\right]$$

$\Rightarrow 1 + abc = 0$

$\Rightarrow abc = -1$

216 **(b)**

$$\begin{vmatrix} x+\omega^2 & \omega & 1 \\ \omega & \omega^2 & 1+x \\ 1 & x+\omega & \omega^2 \end{vmatrix} = 0$$

Applying $C_1 \to C_1 + C_2 + C_3$

$$\begin{vmatrix} x & \omega & 1 \\ x & \omega^2 & 1+x \\ x & x+\omega & \omega^2 \end{vmatrix} = 0 \quad (\therefore 1+\omega+\omega^2 = 0)$$

$\Rightarrow$ $x = 0$ is one of the values of x which satisfy the above determinant equation.

217 **(a)**

We have,

$$|A| = \begin{bmatrix} 4 & 5 & 6 & x \\ 5 & 6 & 7 & y \\ 6 & 7 & 8 & z \\ x & y & z & 0 \end{bmatrix}$$

$$\Rightarrow |A| = \begin{bmatrix} 0 & 0 & 0 & x-2y+z \\ 5 & 6 & 7 & y \\ 6 & 7 & 8 & z \\ x & y & z & 0 \end{bmatrix} \quad \text{Applying } R_1 \to R_1 - 2R_2 + R_3$$

$$\Rightarrow |A| = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 5 & 6 & 7 & y \\ 6 & 7 & 8 & z \\ x & y & z & 0 \end{bmatrix} \quad \left[\begin{matrix} \because x, y, z \text{ are in A. P.} \\ \therefore x - 2y + z = 0 \end{matrix}\right]$$

$\Rightarrow |A| = 0$

218 **(a)**

$$\text{Given, } \Delta = \begin{vmatrix} (e^{i\alpha}+e^{-i\alpha})^2 & (e^{i\alpha}-e^{-i\alpha})^2 & 4 \\ (e^{i\beta}+e^{-i\beta})^2 & (e^{i\beta}-e^{-i\beta})^2 & 4 \\ (e^{i\gamma}+e^{-i\gamma})^2 & (e^{i\gamma}-e^{-i\gamma})^2 & 4 \end{vmatrix}$$

Applying $C_1 \to C_1 - C_2$

$$= \begin{vmatrix} 4 & (e^{i\alpha}-e^{-i\alpha})^2 & 4 \\ 4 & (e^{i\beta}-e^{-i\beta})^2 & 4 \\ 4 & (e^{i\gamma}-e^{-i\gamma})^2 & 4 \end{vmatrix}$$

$= 0$ ($\because$ two columns are same)

Hence, it is independent of α, β and γ.

219 **(b)**

Let A be the first term and R be the common ratio of the GP. Then,

$a = A\,R^{p-1} \Rightarrow \log a = \log A + (p-1)\log R$

$b = A\,R^{q-1} \Rightarrow \log b = \log A + (q-1)\log R$

$c = A\,R^{r-1} \Rightarrow \log c = \log A + (r-1)\log R$

Now,

$$\begin{vmatrix} \log a & p & 1 \\ \log b & q & 1 \\ \log c & r & 1 \end{vmatrix}$$

$$= \begin{vmatrix} (p-1) & \log R & p & 1 \\ (q-1) & \log R & q & 1 \\ (r-1) & \log R & r & 1 \end{vmatrix}$$

$$= \log R = \begin{vmatrix} p-1 & p & 1 \\ q-1 & q & 1 \\ r-1 & r & 1 \end{vmatrix}$$ [Applying $C_1 \to C_1 - (\log A)\, C_3$]

$$= \log R \begin{vmatrix} 0 & p & 1 \\ 0 & q & 1 \\ 0 & r & 1 \end{vmatrix} = 0$$ [Applying $C_1 \to C_1 - C_2 + C_3$]

220 **(c)**

We know that the sum of the products of the elements of a row with the cofactors of the corresponding elements is always equal to the value of the determinant *.ie*, $|A|$.

221 **(d)**

$\because a, b, c, d, e$ and f are in GP.

$\therefore a = a, b = ar, c = ar^2, d = ar^3, e = ar^4$ and $f = ar^5$

$$\therefore \begin{vmatrix} a^2 & d^2 & x \\ b^2 & e^2 & y \\ c^2 & f^2 & z \end{vmatrix} = \begin{vmatrix} a^2 & a^2r^6 & x \\ a^2r^2 & a^2r^8 & y \\ a^2r^4 & a^2r^{10} & z \end{vmatrix}$$

$$= a^4r^6 \begin{vmatrix} 1 & 1 & x \\ r^2 & r^2 & y \\ r^4 & r^4 & z \end{vmatrix} = 0$$

Thus, the given determinant is independent of x, y and z.

222 **(a)**

$$\begin{vmatrix} 1 & \log_x y & \log_x z \\ \log_y x & 1 & \log_y z \\ \log_z x & \log_z y & 1 \end{vmatrix}$$

$= 1(1 - \log_y z \log_z y)$
$- \log_x y(\log_y x - \log_z x \log_y z)$

$+\log_x z\ (\log_z y\ \log_y x - \log\ _z x)$

$= (1 - \log_y y) - \log_x y\ (\log_y x - \log_y x)$

$+\log_x z(\log_z x - \log_z x)$

$= (1 - 1) - 0 + 0 = 0$

223 **(d)**

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1-x & 1 \\ 1 & 1 & 1+y \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 1 & -x & 0 \\ 1 & 0 & y \end{vmatrix} \begin{bmatrix} C_2 \to C_2 - C_1 \\ C_3 \to C_3 - C_1 \end{bmatrix}$$

$= -xy$

224 **(c)**

$$\begin{vmatrix} x & y & z \\ -x & y & z \\ x & -y & z \end{vmatrix} = \begin{vmatrix} x & y & z \\ -x & y & z \\ 0 & 0 & 2z \end{vmatrix} \quad [R_3 \to R_3 + R_2]$$

$= 2z(xy + xy) = 4xyz$

On comparing with $kxyz$, we get $k = 4$

225 **(b)**

Applying $R_1 \to R_1 + R_2 + R_3$ and taking common $(2x + 10)$ from R_1,we get

$$(2x+10) \begin{vmatrix} 1 & 1 & 1 \\ 2 & 2x & 2 \\ 7 & 6 & 2x \end{vmatrix} = 0$$

$$\Rightarrow (2x+10) \begin{vmatrix} 1 & 0 & 0 \\ 2 & 2x-2 & 0 \\ 7 & -1 & 2x-7 \end{vmatrix} = 0$$

$[C_3 \to C_3 - C_1 \text{and } C_2 \to C_2 - C_1]$

$\Rightarrow \ (2x + 10)(2x - 2)(2x - 7) = 0$

$\Rightarrow x = -5, 1, \frac{7}{2}$

Hence , other roots are 1 and $\frac{7}{2}$ or 1 and 3.5

226 **(b)**

Let $\frac{x^2}{a^2} = X, \frac{y^2}{b^2} = Y$ and $\frac{z^2}{c^2} = Z$

Then the given system of equations becomes

$X + Y - Z = 1, X - Y + Z = 1, -X + Y + Z = 1$

The coefficient matrix is $A = \begin{bmatrix} 1 & 1 & -1 \\ 1 & -1 & 1 \\ -1 & 1 & 1 \end{bmatrix}$

Clearly, $|A| \neq 0$. So, the given system of equations has a unique solution

227 **(c)**

Applying $R_1 \to R_1 + R_2$, we get

$$\begin{vmatrix} 2 & 2 & 1 \\ \cos^2\theta & 1+\cos^2\theta & \cos^2\theta \\ 4\sin 4\theta & 4\sin 4\theta & 1+4\sin 4\theta \end{vmatrix} = 0$$

Applying $C_1 \to C_1 - 2C_3, C_2 \to C_2 - 2C_3$

$$\begin{vmatrix} 0 & 0 & 1 \\ -\cos^2\theta & 1-\cos^2\theta & \cos^2\theta \\ -2-4\sin 4\theta & -2-4\sin 4\theta & 1+4\sin 4\theta \end{vmatrix} = 0$$

$\Rightarrow [\cos^2\theta(2 + 4\sin 4\theta) + (1 - \cos^2\theta)(2 + 4\sin 4\theta)] = 0$

$\Rightarrow [2\cos^2\theta + 4\cos^2\theta \sin 4\theta + 2 + 4\sin 4\theta - 2\cos^2\theta - 4\cos^2\theta \sin 4\theta] = 0$

$\Rightarrow 2 + 4\sin 4\theta = 0$

$\Rightarrow \ \sin 4\theta = -\frac{1}{2}$

228 **(a)**

Given determinant, $\Delta \equiv \begin{vmatrix} a & a^2 & a^3+1 \\ b & b^2 & b^3+1 \\ c & c^2 & c^3+1 \end{vmatrix} = 0$

On splitting the determinant into two determinants, we get

$$\Delta \equiv abc \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} + \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = 0$$

$\Rightarrow (1 + abc)[1(bc^2 - cb^2) - a(c^2 - b^2) + a^2(c - b)] = 0$

$\Rightarrow (1 + abc)[(a - b)(b - c)(c - a)] = 0$

Since a, b, c are different, the second factor cannot be zero.

Hence, $1 + abc = 0$

229 **(b)**

We have,

$$\begin{vmatrix} a & a^2-bc & 1 \\ b & b^2-ca & 1 \\ c & c^2-ab & 1 \end{vmatrix}$$

$$= \begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix} + \begin{vmatrix} a & -bc & 1 \\ b & -ca & 1 \\ c & -ab & 1 \end{vmatrix}$$

$$= \begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix}$$

$$+ \frac{1}{abc} \begin{vmatrix} a^2 & -abc & a \\ b^2 & -abc & b \\ c^2 & -abc & c \end{vmatrix}$$ Applying $R_1 \to R_1(a)$ $R_2 \to R_2(b), R_3 \to R_3(c)$ in the IInd determinant

$$= \begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix} - \begin{vmatrix} a^2 & 1 & a \\ b^2 & 1 & b \\ c^2 & 1 & c \end{vmatrix}$$

$$= \begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix} - \begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix} = 0$$

230 **(d)**

Given that, $x^a y^b = e^m, x^c y^d = e^n$

and $\Delta_1 = \begin{vmatrix} m & b \\ n & d \end{vmatrix}, \Delta_2 = \begin{vmatrix} a & m \\ c & n \end{vmatrix}, \Delta_3 = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$

$\Rightarrow a\log x + b\log y = m$

$\Rightarrow c\log x + d\log y = n$

By Cramer's rule

$\log x = \frac{\Delta_1}{\Delta_3}$ and $\log y = \frac{\Delta_2}{\Delta_3}$

$\Rightarrow x = e^{\Delta_1/\Delta_3}$ and $y = e^{\Delta_2/\Delta_3}$

231 **(d)**

Clearly, $x = 0$ satifies the given equation

232 **(c)**

Let $\Delta = \begin{vmatrix} 10! & 11! & 12! \\ 11! & 12! & 13! \\ 12! & 13! & 14! \end{vmatrix}$

$= 10!\,11!\,12! \begin{vmatrix} 1 & 11 & 11\times 12 \\ 1 & 12 & 12\times 13 \\ 1 & 13 & 13\times 14 \end{vmatrix}$

Applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$= 10!\,11!\,12! \begin{vmatrix} 1 & 11 & 11\times 12 \\ 0 & 1 & 24 \\ 0 & 2 & 50 \end{vmatrix}$

$= (10!\,11!\,12!)(50-48)$

$= 2 \cdot (10!\,11!\,12!)$

233 **(c)**

We have, $\begin{vmatrix} \sin x & \cos x & \cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix} = 0$

Applying $C_1 \to C_1 + C_2 + C_3$

$\Rightarrow \begin{vmatrix} \sin x + 2\cos x & \cos x & \cos x \\ \sin x + 2\cos x & \sin x & \cos x \\ \sin x + 2\cos x & \cos x & \sin x \end{vmatrix} = 0$

$\Rightarrow (2\cos x + \sin x) \begin{vmatrix} 1 & \cos x & \cos x \\ 1 & \sin x & \cos x \\ 1 & \cos x & \sin x \end{vmatrix} = 0$

Applying $R_2 \to R_2 - R_1, R_3 \to R_3 - R_1$

$\Rightarrow (2\cos x + \sin x) \begin{vmatrix} 1 & \cos x & \cos x \\ 0 & \sin x - \cos x & 0 \\ 0 & 0 & \sin x - \cos x \end{vmatrix} = 0$

$\Rightarrow (2\cos x + \sin x)(\sin x - \cos x)^2 = 0$

$\therefore \tan x = -2, 1$ But $\tan x \neq -2$, because it does not lie in the interval $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$.

$\therefore \tan x = 1$

So, $x = \frac{\pi}{4}$

234 **(a)**

$\begin{vmatrix} (a^x + a^{-x})^2 & (a^x - a^{-x})^2 & 1 \\ (b^x + b^{-x})^2 & (b^x - b^{-x})^2 & 1 \\ (c^x + c^{-x})^2 & (c^x - c^{-x})^2 & 1 \end{vmatrix}$

Applying $C_1 \to C_1 - C_2$

$= \begin{vmatrix} 4 & (a^x - a^{-x})^2 & 1 \\ 4 & (b^x - b^{-x})^2 & 1 \\ 4 & (c^x - c^{-x})^2 & 1 \end{vmatrix} =$

$4 \begin{vmatrix} 1 & (a^x - a^{-x})^2 & 1 \\ 1 & (b^x - b^{-x})^2 & 1 \\ 1 & (c^x - c^{-x})^2 & 1 \end{vmatrix} = 0$ ($\because$two columns are identical)

235 **(c)**

Given matrix is non-singular, then

$\begin{vmatrix} \lambda & 1 & 0 \\ 0 & 2 & 3 \\ 0 & 0 & \lambda \end{vmatrix} \neq 0$

$\Rightarrow \lambda(2\lambda - 0) \neq 0$

$\Rightarrow \lambda \neq 0$

236 **(d)**

Let $\Delta = \begin{vmatrix} a^2 & b^2 & c^2 \\ (a+1)^2 & (b+1)^2 & (c+1)^2 \\ (a-1)^2 & (b-1)^2 & (c-1)^2 \end{vmatrix}$

Applying $R_2 \to R_2 - R_3$

$= \begin{vmatrix} a^2 & b^2 & c^2 \\ 4a & 4b & 4c \\ (a-1)^2 & (b-1)^2 & (c-1)^2 \end{vmatrix}$

$= 4 \begin{vmatrix} a^2 & b^2 & c^2 \\ a & b & c \\ (a-1)^2 & (b-1)^2 & (c-1)^2 \end{vmatrix}$

Applying $R_3 \to R_3 - (R_1 - 2R_2)$

$= 4 \begin{vmatrix} a^2 & b^2 & c^2 \\ a & b & c \\ 1 & 1 & 1 \end{vmatrix}$

$\therefore k = 4$

237 **(c)**

Let $f(x) = a_0x^2 + a_1x + a_2$

and $g(x) = b_2x^2 + b_1x + b_2$

Also, $h(x) = c_0x^2 + c_1x + c_2$

Then, $\Delta(x) = \begin{vmatrix} f(x) & g(x) & h(x) \\ 2a_0x + a_1 & 2b_0x + b_1 & 2c_0x + c_1 \\ 2a_0 & 2b_0 & 2c_0 \end{vmatrix}$

$= x \begin{vmatrix} f(x) & g(x) & h(x) \\ 2a_0 & 2b_0 & 2c_0 \\ 2a_0 & 2b_0 & 2c_0 \end{vmatrix} + \begin{vmatrix} f(x) & g(x) & h(x) \\ a_1 & b_1 & c_1 \\ 2a_0 & 2b_0 & 2c_0 \end{vmatrix}$

$= 0 + 2 \begin{vmatrix} f(x) & g(x) & h(x) \\ a_1 & b_1 & c_1 \\ a_0 & b_0 & c_0 \end{vmatrix}$

$= 2[(b_1c_0 - b_0c_1)f(x) - (a_1c_0 - a_0c_1)g(x) + (a_1b_0 - a_0b_1)h(x)]$

Hence, degree of $\Delta(x) \leq 2$

238 **(d)**

Applying $C_1 \to C_1 + C_2 + C_3$, we get

$\begin{vmatrix} 2(x+y+z) & y+z & z+x \\ x+y+z & y & z \\ 0 & y-z & z-x \end{vmatrix}$

$= (x+y+z) \begin{vmatrix} 2 & y-z & z+x \\ 1 & y & z \\ 0 & y-z & z-x \end{vmatrix}$

Applying $R_2 \to 2R_2 - R_1$

$= (x+y+z) \begin{vmatrix} 2 & y+z & z+x \\ 0 & y-z & z-x \\ 0 & y-z & z-x \end{vmatrix}$

$= 0$ [$\therefore$ two rows are identical]

239 **(d)**

Given, $f(x) = \begin{vmatrix} 1+a & 1+ax & 1+ax^2 \\ 1+b & 1+bx & 1+bx^2 \\ 1+b & 1+cx & 1+cx^2 \end{vmatrix}$

$\Rightarrow f(x) = \begin{vmatrix} 1+a & a(x-1) & ax(x-1) \\ 1+b & b(x-1) & bx(x-1) \\ 1+b & c(x-1) & cx(x-1) \end{vmatrix}$

$= (x-1)x(x-1) \begin{vmatrix} 1+a & a & a \\ 1+b & b & b \\ 1+c & c & c \end{vmatrix} = 0$

($\therefore$ two columns are same)

240 **(c)**

We have,

$ax^4 + bx^3 + cx^2 + 50x + d$

$$= \begin{vmatrix} x^3 - 14x^2 & -x & 3x + \lambda \\ 4x + 1 & 3x & x - 4 \\ -3 & 4 & 0 \end{vmatrix}$$

On differentiating with respect to x, we get

$4ax^3 + 3bx^2 + 2cx + 50$

$$= \begin{vmatrix} 3x^2 - 28x & -1 & 3 \\ 4x + 1 & 3x & x - 4 \\ -3 & 4 & 0 \end{vmatrix} + \begin{vmatrix} x^3 - 14x^2 & -x & 3x + \lambda \\ 4 & 3 & 1 \\ -3 & 4 & 0 \end{vmatrix}$$

Now, put $x = 0$, we get

$$50 = \begin{vmatrix} 0 & -1 & 3 \\ 1 & 0 & -4 \\ -3 & 4 & 0 \end{vmatrix} + \begin{vmatrix} 0 & 0 & \lambda \\ 4 & 3 & 1 \\ -3 & 4 & 0 \end{vmatrix}$$

$\Rightarrow 50 = 25\lambda$

$\Rightarrow \lambda = 2$

241 **(d)**

We have, $\begin{vmatrix} x^2 + x & x + 1 & x - 2 \\ 2x^2 + 3x - 1 & 3x & 3x - 3 \\ x^2 + 2x + 3 & 2x - 1 & 2x - 1 \end{vmatrix} = Ax - 12$

On putting $x = 1$ on both sides, we get

$$\begin{vmatrix} 2 & 2 & -1 \\ 4 & 3 & 0 \\ 6 & 1 & 1 \end{vmatrix} = A - 12$$

Applying $C_1 \to C_1 - C_2$

$$\Rightarrow \begin{vmatrix} 0 & 2 & -1 \\ 1 & 3 & 0 \\ 5 & 1 & 1 \end{vmatrix} = A - 12$$

$\Rightarrow -2(1) + (-1)(-14) = A - 12$

$\Rightarrow A = 24$

242 **(a)**

We have, $\begin{vmatrix} x + \alpha & \beta & \gamma \\ \gamma & x + \beta & \alpha \\ \alpha & \beta & x + \gamma \end{vmatrix} = 0$

Applying $C_1 \to C_1 + C_2 + C_3$

$$\Rightarrow \begin{vmatrix} x + \alpha + \beta + \gamma & \beta & \gamma \\ x + \alpha + \beta + \gamma & x + \beta & \alpha \\ x + \alpha + \beta + \gamma & \beta & x + \gamma \end{vmatrix} = 0$$

$$\Rightarrow (x + \alpha + \beta + \gamma) \begin{vmatrix} 1 & \beta & \gamma \\ 1 & x + \beta & \alpha \\ 1 & \beta & x + \gamma \end{vmatrix} = 0$$

Applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$$\Rightarrow (x + \alpha + \beta + \gamma) \begin{vmatrix} 1 & \beta & \gamma \\ 0 & x & \alpha - \gamma \\ 0 & 0 & x \end{vmatrix} = 0$$

$\Rightarrow (x + \alpha + \beta + \gamma)(x^2 - 0) = 0$

$\Rightarrow x = 0$ or $x = -(\alpha + \beta + \gamma)$

243 **(b)**

We have,

$$\Delta = \begin{vmatrix} 1/a & 1 & bc \\ 1/b & 1 & ca \\ 1/c & 1 & ab \end{vmatrix}$$

$\Rightarrow \Delta$

$$= \frac{1}{abc} \begin{vmatrix} 1 & a & abc \\ 1 & b & abc \\ 1 & c & abc \end{vmatrix}$$ Applying $R_1 \to R_1(a)$, $R_2 \to R_2(b)$ and $R_3 \to R_3(c)$

$$\Rightarrow \Delta = \frac{abc}{abc} \begin{vmatrix} 1 & a & 1 \\ 1 & b & 1 \\ 1 & c & 1 \end{vmatrix}$$ [Taking abc common from C_3]

$\Rightarrow \Delta = \frac{abc}{abc} \times 0 = 0$

244 **(b)**

We have, $|A| \neq 0$. Therefore, A^{-1} exists

Now, $AB = AC$

$\Rightarrow A^{-1}(AB) = A^{-1}(AC)$

$\Rightarrow (A^{-1}A)B = (A^{-1}A)C \Rightarrow B = C$

246 **(c)**

Applying $C_3 \to C_3 - \omega\, C_1$, we get

$$\begin{vmatrix} a & b\omega^2 & a\omega \\ b\omega & c & b\omega^2 \\ c\omega^2 & a\omega & c \end{vmatrix} = \begin{vmatrix} a & b\omega^2 & 0 \\ b\omega & c & 0 \\ c\omega^2 & a\omega & 0 \end{vmatrix} = 0$$

247 **(d)**

$$\begin{vmatrix} a + b & a + 2b & a + 3b \\ a + 2b & a + 3b & a + 4b \\ a + 4b & a + 5b & a + 6b \end{vmatrix}$$

$$= \begin{vmatrix} a + b & a + 2b & a + 3b \\ b & b & b \\ 2b & 2b & 2b \end{vmatrix} \quad \begin{pmatrix} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_2 \end{pmatrix}$$

$= 0$ ($\because$ R_2 and R_3 are proportional)

248 **(c)**

Applying $R_1 \to R_1 - (R_2 + R_3)$, we get

$$\begin{vmatrix} 0 & -2z & -2y \\ y & z + x & y \\ z & z & x + y \end{vmatrix}$$

$= 2z(xy + y^2 - yz) - 2y(yz - z^2 - xz)$

$= 2xyz + 2y^2z - 2yz^2 - 2y^2z + 2yz^2 + 2xyz$

$= 4xyz$

249 **(b)**

We have,

$$\frac{d}{dx}(\Delta_1) = \begin{vmatrix} 1 & 0 & 0 \\ a & x & b \\ a & a & x \end{vmatrix} + \begin{vmatrix} x & b & b \\ 0 & 1 & 0 \\ a & a & x \end{vmatrix} + \begin{vmatrix} x & b & b \\ a & x & b \\ 0 & 0 & 1 \end{vmatrix}$$

$$\Rightarrow \frac{d}{dx}(\Delta_1) = \begin{vmatrix} x & b \\ a & x \end{vmatrix} + \begin{vmatrix} x & b \\ a & x \end{vmatrix} + \begin{vmatrix} x & b \\ a & x \end{vmatrix} = 3\,\Delta_2$$

251 **(d)**

Applying $C_2 \to C_2 - C_1, C_3 \to C_3 - C_2$, we get

$$\begin{vmatrix} 1990 & 1 & 1 \\ 1991 & 1 & 1 \\ 1992 & 1 & 1 \end{vmatrix} = 0$$

253 **(b)**

We have,

Δ

$$= \begin{vmatrix} 1+a_1b_1+a_1^2b_1^2 & 1+a_1b_2+a_1^2b_2^2 & 1+a_1b_3 \\ 1+a_2b_1+a_2^2b_1^2 & 1+a_2b_2+a_2^2b_2^2 & 1+a_2b_3 \\ 1+a_3b_1+a_3^2b_1^2 & 1+a_3b_2+a_3^2b_2^2 & 1+a_2b_3 \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} 1 & a_1 & a_1^2 \\ 1 & a_2 & a_2^2 \\ 1 & a_3 & a_3^2 \end{vmatrix} \begin{vmatrix} 1 & b_1 & b_1^2 \\ 1 & b_2 & b_2^2 \\ 1 & b_3 & b_3^2 \end{vmatrix}$$

$$\Rightarrow \Delta = (a_1-a_2)(a_2-a_3)(a_3-a_1)(b_1-b_2)(b_2-b_3)(b_3-b_1)$$

254 **(b)**

Let $A \equiv \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 5$ ...(i)

$$\therefore \begin{vmatrix} b_2c_3-b_3c_2 & c_2a_3-c_3a_2 & c_2b_3-c_3b_2 \\ b_3c_1-b_1c_3 & c_3a_1-c_1a_3 & a_3b_1-a_1b_3 \\ b_1c_2-b_2c_1 & c_1a_2-c_2a_1 & a_1b_2-a_2b_1 \end{vmatrix}$$

$|\text{adj } A| = (5)^{3-1}$ [from Eq. (i)]

$= 5^2 = 25$ $(\because |\text{adj } A| = |A|^{n-1})$

255 **(b)**

Let $a \neq 0$. Then,

$$\Delta = \frac{1}{a}\begin{vmatrix} a^3+ax^2 & ab & ac \\ a^2b & b^2+x^2 & bc \\ a^2c & bc & c^2+x^2 \end{vmatrix}$$ [Applying $C_1 \to a\,C_1$]

$$\Rightarrow \Delta = \frac{1}{a}\begin{vmatrix} a(a^2+b^2+c^2+x^2) & ab & ac \\ b(a^2+b^2+c^2+x^2) & b^2+x^2 & bc \\ c(a^2+b^2+c^2+x^2) & bc & c^2+x^2 \end{vmatrix}$$

[Applying $C_1 \to C_1 + b\,C_2 + c\,C_3$]

$$\Rightarrow \Delta = \frac{1}{a}(a^2+b^2+c^2+x^2)\begin{vmatrix} a & ab & ac \\ b & b^2+x^2 & bc \\ c & bc & c^2+x^2 \end{vmatrix}$$

$$\Rightarrow \Delta = \frac{1}{a}(a^2+b^2+c^2+x^2)\begin{vmatrix} a & 0 & 0 \\ b & x^2 & 0 \\ c & 0 & x^2 \end{vmatrix},$$

[Applying $C_2 \to C_2 - b\,C_1, C_3 \to C_3 - c\,C_1$]

$\Rightarrow \Delta = (a^2+b^2+c^2+x^2)x^4$

Clearly, Δ is divisible by x^4

If $a = 0$, then also it can be easily seen that Δ is divisible by x^4

256 **(a)**

We have,

$$\Delta_a = \begin{vmatrix} a-1 & 2 & 6 \\ (a-1)^2 & 2n^2 & 4n-2 \\ (a-1)^3 & 3n^3 & 2n^2-3n \end{vmatrix}$$

$$\therefore \sum_{a=1}^{n}\Delta_a \begin{vmatrix} \sum_{a=1}^{n}(a-1) & n & 6 \\ \sum_{a=1}^{n}(a-1)^2 & 2n^2 & 4n-2 \\ \sum_{a=1}^{n}(a-1)^3 & 3n^2 & 3n^2-3n \end{vmatrix}$$

$$\Rightarrow \sum_{a=1}^{n}\Delta_a = \begin{vmatrix} \frac{n(n-1)}{2} & n & 6 \\ \frac{n(n-1)(2n-1)}{6} & 2n^2 & 4n-2 \\ \left(\frac{n(n-1)}{2}\right)^2 & 3n^3 & 3n^2-3n \end{vmatrix}$$

$$\Rightarrow \sum_{a=1}^{n}\Delta_a = \frac{n(n-1)}{12}\begin{vmatrix} 6 & n & 6 \\ 4n-2 & 2n^2 & 4n-2 \\ 3n^2-3n & 3n^3 & 3n^2-3n \end{vmatrix} = 0$$

257 **(d)**

$B = 5A^2$

$\Rightarrow \det(B) = \det(5A^2) = 5^3[\det(A)]^2$

$= 125(6)^2 = 4500$ [given $\det A = 6$]

258 **(b)**

Given, $f(x) = \begin{vmatrix} x & 1+\sin x & \cos x \\ 1 & \log(1+x) & 2 \\ x^2 & 1+x^2 & 0 \end{vmatrix}$

$= x\{-2(1+x^2)\} - (1+\sin x)(-2x^2) + \cos x\{1+x^2-x^2\log(1+x)\}$

$= -2x - 2x^3 + 2x^2 + 2x^2\sin x + \cos x\{1+x^2-x^2\log(1+x)\}$

$\therefore$ Coefficient of x in $f(x) = -2$

259 **(c)**

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ bc & ca & ab \\ b+c & c+a & a+b \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 0 & 0 \\ bc & c(a-b) & a(b-c) \\ b+c & (a-b) & (b-c) \end{vmatrix} \begin{bmatrix} C_2 \to C_2 - C_1 \\ C_3 \to C_3 - C_2 \end{bmatrix}$$

$$= (a-b)(b-c)\begin{vmatrix} 1 & 0 & 0 \\ bc & c & a \\ b+c & 1 & 1 \end{vmatrix}$$

$= (a-b)(b-c)(c-a)$

260 **(b)**

Since, $\Delta(1) = 0$ and $\Delta'(1) = 0$ so, $(x-1)^2$ is a factor of $\Delta(x)$

261 **(d)**

On putting $\lambda = 0$, we get

$t = \begin{vmatrix} b^2+c^2 & a^2 & a^2 \\ b^2 & c^2+a^2 & b^2 \\ c^2 & c^2 & a^2+b^2 \end{vmatrix} = 4a^2b^2c^2$

Clearly, it depends on a, b, c.

262 **(c)**

Let $\Delta = \begin{vmatrix} 10! & 11! & 12! \\ 11! & 12! & 13! \\ 12! & 13! & 14! \end{vmatrix}$

$= (10!)(11!)(12!) \begin{vmatrix} 1 & 11 & 11\times 12 \\ 1 & 12 & 12\times 13 \\ 1 & 13 & 13\times 14 \end{vmatrix}$

$= (10!)(11!)(12!) \begin{vmatrix} 1 & 11 & 11\times 12 \\ 0 & 1 & 24 \\ 0 & 2 & 50 \end{vmatrix}$

$= 2(10!)(11!)(12!)$

263 **(b)**

$\because \det(A_1) = \begin{vmatrix} a & b \\ b & a \end{vmatrix} = a^2 - b^2$

$\det(A_2) = \begin{vmatrix} a^2 & b^2 \\ b^2 & a^2 \end{vmatrix} = a^4 - b^4$

$\therefore \sum_{i=1}^{\infty} \det(A_i) = \det(A_1) + \det(A_2) + \dots$

$= a^2 - b^2 + a^4 - b^4 + \dots$

$= \frac{a^2}{1-a^2} - \frac{b^2}{1-b^2} = \frac{a^2-b^2}{(1-a^2)(1-b^2)}$

264 **(c)**

Since, A is a singular matrix

$\therefore |A| = 0$

$\Rightarrow \begin{bmatrix} 1 & 2 & x \\ 4 & -1 & 7 \\ 2 & 4 & -6 \end{bmatrix} = 0$

$\Rightarrow 1(6-28) - 2(-24-14) + x[16+2] = 0$

$\Rightarrow -22 + 76 + 18x = 0 \Rightarrow x = -3$

265 **(b)**

$\begin{vmatrix} x & p & q \\ p & x & q \\ p & q & x \end{vmatrix} = \begin{vmatrix} x+p+q & p & q \\ x+p+q & x & q \\ x+p+q & q & x \end{vmatrix}$

$= (x+p+q) \begin{vmatrix} 1 & p & q \\ 1 & x & q \\ 1 & q & x \end{vmatrix}$

$= (x+p+q) \begin{vmatrix} 1 & p & q \\ 0 & x-p & 0 \\ 0 & q-p & x-q \end{vmatrix}$

$= (x+p+q) \begin{bmatrix} x-p & 0 \\ q-p & x-q \end{bmatrix}$

$= (x-p)(x-q)(x+p+q)$

266 **(b)**

We have, $\begin{vmatrix} 1 & 4 & 20 \\ 1 & -2 & 5 \\ 1 & 2x & 5x^2 \end{vmatrix} = 0$

$\Rightarrow \begin{vmatrix} 0 & 6 & 15 \\ 0 & -2-2x & 5(1-x^2) \\ 1 & 2x & 5x^2 \end{vmatrix} = 0 \quad \begin{pmatrix} R_1 \to R_1 - R_2 \\ \text{and } R_2 \to R_2 - R_3 \end{pmatrix}$

$\Rightarrow 3\cdot 2\cdot 5 \begin{vmatrix} 0 & 1 & 1 \\ 0 & -(1+x) & 1-x^2 \\ 1 & x & x^2 \end{vmatrix} = 0$

(Taking common, 3 from R_1, 2 from C_2, 5 from C_3)

$\Rightarrow (1+x) \begin{vmatrix} 0 & 1 & 1 \\ 0 & -1 & 1-x \\ 1 & x & x^2 \end{vmatrix} = 0$

$\Rightarrow (1+x)(2-x) = 0$

$\Rightarrow x+1 = 0$ or $x-2 = 0 \Rightarrow x = -1, 2$

267 **(d)**

$x + iy = -3i \begin{vmatrix} 6i & 1 & 1 \\ 4 & -1 & -1 \\ 20 & i & i \end{vmatrix} = 0$

$\Rightarrow x = 0, \quad y = 0$

268 **(a)**

We have,

$\Delta = \begin{vmatrix} \cos(\alpha+\beta) & -\sin(\alpha+\beta) & \cos 2\beta \\ \sin\alpha & \cos\alpha & \sin\beta \\ -\sin\alpha & \sin\alpha & \cos\beta \end{vmatrix}$

$\Rightarrow \Delta = \begin{vmatrix} 0 & 0 & \cos 2\beta + 1 \\ \sin\alpha & \cos\alpha & \sin\beta \\ -\cos\alpha & \sin\alpha & \cos\beta \end{vmatrix}$

$\begin{bmatrix} \text{Applying } R_1 \to R_1 + R_2 \\ \sin\beta + R_3\cos\beta \end{bmatrix}$

$\Rightarrow \Delta = (\cos 2\beta + 1)(\sin^2\alpha + \cos^2\alpha)$

$= \cos 2\beta + 1,$

Which is independent of α

269 **(d)**

Given $\begin{vmatrix} x+1 & x+2 & x+a \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix} = 0$

Applying $R_1 \to +R_1 + R_3 - 2R_2$, we get

$\begin{vmatrix} 0 & 0 & a+c-2b \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix} = 0$

$\Rightarrow (a+c-2b)[x^2+6x+8-(x^2+6x+9)] = 0$

$\Rightarrow (a+c-2b)(-1) = 0$

$\Rightarrow 2b = a+c$

$\Rightarrow a, b, c$ are in AP

270 **(a)**

We have,

$\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix}$

$= abc \begin{vmatrix} 1+\frac{1}{a} & \frac{1}{a} & \frac{1}{a} \\ \frac{1}{b} & 1+\frac{1}{b} & \frac{1}{b} \\ \frac{1}{c} & \frac{1}{c} & 1+\frac{1}{c} \end{vmatrix}$ Applying $R_1 \to R_1$ $R_2 \to R_2\left(\frac{1}{b}\right), R_3 \to$

$$= abc\begin{vmatrix} 1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c} & 1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c} & 1+\frac{1}{a}+\frac{1}{b} \\ \frac{1}{b} & 1+\frac{1}{b} & \frac{1}{b} \\ \frac{1}{c} & \frac{1}{c} & 1+\frac{1}{c} \end{vmatrix}$$

Applying $R_1 \to R_1 + R_2 + R_3$

$$= abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)\begin{vmatrix} 1 & 1 & 1 \\ \frac{1}{b} & 1+\frac{1}{b} & \frac{1}{b} \\ \frac{1}{c} & \frac{1}{c} & 1+\frac{1}{c} \end{vmatrix}$$

$$= abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)\begin{vmatrix} 1 & 0 & 0 \\ \frac{1}{b} & 1 & 0 \\ \frac{1}{c} & 0 & 1 \end{vmatrix} \quad \begin{bmatrix}\text{Applying } C_2 \to C_2 - C_1 \\ C_3 \to C_3 - C_1\end{bmatrix}$$

$$= abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$$

271 **(a)**

On putting $x = 0$,we observe that the determinant becomes zero.

$$\therefore \Delta = \begin{vmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{vmatrix}$$

$= a(bc) - b(ac) = 0$

Hence, $x = 0$ is a root of the given equation

272 **(a)**

$$\sum_{r=0}^{n} D_r = \begin{vmatrix} \sum r & 1 & \frac{n(n+1)}{2} \\ 2\sum r - \sum 1 & 4 & n^2 \\ \sum 2^{r-1} & 5 & 2^n - 1 \end{vmatrix}$$

$$= \begin{vmatrix} \frac{n(n+1)}{2} & 1 & \frac{n(n+1)}{2} \\ n^2 & 4 & n^2 \\ 2^n - 1 & 5 & 2^n - 1 \end{vmatrix} = 0$$

[$\because$ two columns are identical]

273 **(b)**

Given, $f(\alpha) = \begin{vmatrix} 1 & \alpha & \alpha^2 \\ \alpha & \alpha^2 & 1 \\ \alpha^2 & 1 & \alpha \end{vmatrix}$

$= 1(\alpha^3 - 1) - \alpha(\alpha^2 - \alpha^2) + \alpha^2(\alpha - \alpha^4)$

$= \alpha^3 - 1 - 0 + \alpha^3 - \alpha^6$

$\Rightarrow f(\sqrt[3]{3} = 3 - 1 - 0 + 3 - 3^2 = 6 - 10 = -4$

274 **(c)**

We have,

$$\Delta = \begin{vmatrix} 2a_1b_1 & a_1b_2 + a_2b_1 & a_1b_3 + a_3b_1 \\ a_1b_2 + a_2b_1 & 2a_2b_2 & a_2b_3 + a_3b_2 \\ a_1b_3 + a_3b_1 & a_3b_2 + a_2b_3 & 2a_3b_3 \end{vmatrix}$$

This can be written as

$$\begin{vmatrix} a_1 & b_1 & 0 \\ a_2 & b_2 & 0 \\ a_3 & b_3 & 0 \end{vmatrix}\begin{vmatrix} b_1 & a_1 & 0 \\ b_2 & a_2 & 0 \\ b_3 & a_3 & 0 \end{vmatrix} = 0$$

275 **(a)**

$a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13}$

$= 3\begin{vmatrix} 2 & 1 \\ 2 & 6 \end{vmatrix} - 2\begin{vmatrix} 1 & 1 \\ 3 & 6 \end{vmatrix} + 4\begin{vmatrix} 1 & 2 \\ 3 & 2 \end{vmatrix}$

$=3(12 - 2) - 2(6 - 3) + 4(2 - 6)$

$= 30 - 6 - 16 = 8$

276 **(c)**

We have,

$$\begin{vmatrix} x-a & x-b & x-c \\ x-b & x-c & x-a \\ x-c & x-a & x-b \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 3x-(a+b+c) & x-b & x-c \\ 3x-(a+b+c) & x-c & x-a \\ 3x-(a+b+c) & x-a & x-b \end{vmatrix} = 0$$

[Applying $C_1 \to C_1 + C_2 + C_3$

$$\Rightarrow \{3x-(a+b+c)\}\begin{vmatrix} 1 & x-b & x-c \\ 1 & x-c & x-a \\ 1 & x-a & x-b \end{vmatrix} = 0$$

$$\Rightarrow \{3x-(a+b+c)\}\begin{vmatrix} 1 & x-b & x-c \\ 0 & b-c & c-a \\ 0 & b-a & c-b \end{vmatrix} = 0$$

$\Rightarrow \{3x-(a+b+c)\}(a^2 + b^2 + c^2 - ab - bc - ca) = 0$

$\Rightarrow x = \frac{1}{3}(a+b+c)$ [$\because a^3 + b^2 + c^2 - ab - bc - ca \neq 0$]

277 **(b)**

Applying $C_1 \to C_1 + C_2 + C_3$, we obtain

$$\begin{vmatrix} x & p & q \\ p & x & q \\ q & q & x \end{vmatrix}$$

$$= \begin{vmatrix} x+p+q & p & q \\ x+p+q & x & q \\ c+p+q & q & x \end{vmatrix}$$

$$= (x+p+q)\begin{vmatrix} 1 & p & q \\ 1 & x & q \\ 1 & q & x \end{vmatrix}$$

$$= (x+p+q)\begin{vmatrix} 1 & p & q \\ 0 & x-p & 0 \\ 0 & q-p & x-q \end{vmatrix}$$

$\begin{bmatrix}\text{Applying } R_2 \to R_2 - R_1, \\ R_3 \to R_3 - R_1\end{bmatrix}$

$= (x+p+q)(x-p)(x-q)$ [Expanding along C_1]

278 **(a)**

Let $f(x) =$

$$\begin{vmatrix} 1 & 1 & 1 \\ 2x & (x-1) & x \\ 3x(x-1) & (x-1)(x-2) & x(x-1) \end{vmatrix}$$

$$= (x-1)\begin{vmatrix} 1 & 1 & 1 \\ 2x & x-1 & x \\ 3x & x-2 & x \end{vmatrix}$$

Applying $C_1 \rightarrow C_1 - C_3$ and $C_2 \rightarrow C_2 - C_3$

$$= (x-1)\begin{vmatrix} 0 & 0 & 1 \\ x & -1 & x \\ 2x & -2 & x \end{vmatrix}$$

$= (x-1)[-2x+2x] = 0$

$\therefore f(x) = 0$

$\Rightarrow f(50) = 0$